Safety Hazards
Fluid Machinery Laboratory Room B-10

HAZARD: Rotating Equipment
Be aware of pinch points and possible entanglement

**Personal Protective Equipment**: Safety Goggles; Standing Shields, Sturdy Shoes
No: Loose clothing; Neck Ties/Scarves; Jewelry (remove); Long Hair (tie back)

HAZARD: Projectiles / Ejected Parts
Articles in motion may dislodge and become airborne

**Personal Protective Equipment**: Safety Goggles; Standing Shields

HAZARD: Electrical - Burn / Shock
Care with electrical connections, particularly with grounding, and not Using frayed electrical cords, can reduce hazard. Use GFCI receptacles near water.

HAZARD: High Pressure Air-Fluid / Gas Cylinders / Vacuum
Inspect system integrity before operating any pressure / vacuum equipment. Gas cylinders must be secured at all times.

**Personal Protective Equipment**: Safety Goggles

HAZARD: Water / Slip Hazard
Clean any spills immediately.

HAZARD: Laser / Eye - Cornea Damage
Do not look directly into laser

**Personal Protective Equipment**: Laser Specific Goggles
Experiment No. 1

PRESSURE DISTRIBUTION AND DRAG FORCE IN FLOW PAST A CYLINDER

I. OBJECTIVE

The object of this experiment is to investigate the pressure distribution, drag and separation point for flow past a cylinder, see Fig. 1. In this experiment, the analytical pressure distribution will be compared with the measured one. The form drag force, due to the pressure distribution will be calculated by integration of the measured pressure. This drag will be compared to the shear force (skin friction) due to the shear stress around the cylinder. The theoretical shear stress distribution is used for this purpose.

![Diagram of flow past a circular cylinder](image)

Figure 1-1: Flow past a circular cylinder

II. BACKGROUND

Hydrodynamic force results from a relative motion between a solid body and a fluid. For viscous fluid, the total force can be divided into two components, i.e. one part caused by the shear stress and the other part by the pressure. The force induced by the shear stresses results from viscous resistance (viscous friction) and it is proportional to the fluid viscosity. The shear force is also referred to as "skin-friction drag". The second part of drag force is form drag that results from non-symmetrical pressure distribution around the cylinder. The pressure drag is independent of the viscosity. It is generated by the inertia of a flowing fluid and its impact on the immersed body. Pressure distribution is generated around the immersed body with resultant form drag force. The form drag is the dominant drag force for high velocity and low viscosity flow (high Re) past large bodies such as air resistance of cars or airplanes. At very low Reynolds numbers, (Re < 1) the shear stress distribution, (due to the viscosity of the fluid) is the dominant force. As the Reynolds number increases, the pressure distribution caused by the inertia of the fluid becomes a more significant factor when
compared to the viscous force. In this experiment, you are expected to demonstrate that this trend of the magnitude of the drag forces components versus Reynolds number is correct.

The drag force components can be determined analytically by integration of shear stress or measured pressure distribution over the surface of the immersed body:

\[
\text{(Total drag force)} = F = \int_A \tau \, dA + \int_A p \, dA
\]

Shear Force Pressure Force

The total hydrodynamic force, \( F \), on the immersed body can also be divided into two components according to the direction, parallel and perpendicular to the direction of flow. The following are these two components:

- \( F_D \) - Drag force, acting in a direction parallel to the mainstream flow,
- \( F_L \) - Lift force, acting in a direction normal to the mainstream flow.

In this experiment, the flow is past a symmetrical circular cylinder. Due to symmetrical pressure and shear distribution around horizontal centerline, there is no resultant lift force, there is only drag force in the flow direction. However for a cross section of half a cylinder, there is lift as well as drag force components.

The equation for the drag force is

\[
F_D = C_D \frac{\rho V^2 A}{2}
\]  \hspace{1cm} (1.2)

where
- \( \rho \) is the density of the flowing fluid
- \( V = U_\infty \) is the main stream velocity. In a flow past a cylinder the velocity \( V \) is referred to as \( U_\infty \) to mathematically indicate that the main flow velocity is far from the cylinder and not affected by it.
- \( A \) is the projection area of body on a plane perpendicular to the flow direction.
- \( C_D \) - drag force coefficient, which is usually determined experimentally

If compressibility is neglected, \( C_D \) is then only a function of the Reynolds number.

\[
\text{Re} = \frac{VD}{\nu}
\]  \hspace{1cm} (1.3)

where \( D \) is the diameter for a cylinder or sphere (for other shapes it is replaced by a characteristic length, \( L \), of the section area) and \( \nu \) is kinematic viscosity, \( \nu = \mu / \rho \). \( \mu \) is the viscosity, and \( \rho \) is the density.
III. EQUIPMENT

The equipment to obtain the above objective is a subsonic wind tunnel with a 14 x 10-in. rectangular working section (throat) where a vertical cylinder OD =3 inch is mounted. The cylinder has holes, at interval of 5 Deg., connected to flexible tubes, leading to multi-port manometer. Air-speed indicator and Pitot tube are provided to measure the velocity in the working section of the wind tunnel.

IV. 2 Pitot Tube

![Diagram of Pitot Tube]

Figure 1-2: Pitot Tube

The Pitot tube, a simple device to measure fluid velocity, was invented by Pitot in the eighteenth century. It consists of L shape, curved tube inside another tube as shown in Fig. 1-2. On the inlet side, the internal tube is open at the end, while the external tube has several small ports around the tube. On the other side the pressures of the internal and external tubes are measured. The inlet centerline is inserted parallel to the main stream velocity. The fluid velocity stagnates inside the internal tube and the velocity head converts to pressure head, while the ports around the external tube sense the static pressure, where the main stream velocity is nearly undisturbed.

Fig. 1-2 shows the following points:
1. is along the stream line along the centerline of the open tube,
2. is the open port of the internal tube where the flow is stagnated and the pressure is elevated to stagnation pressure,
3. are the open ports around the external tube, where there is static pressure,
4. is where the stagnation pressure, \(P_{\text{stagnation}}\), is sensed, and
5. is where the static pressure, \(P_{\text{static}}\), is sensed.

The principle of the Pitot tube is the assumption that outside the tube, near the ports 3, the main-stream velocity is not significantly disturbed by the tube, and the static pressure \(P_{\text{static}}\) in point 3 is very close to \(P_{\text{static}}\) of point 1.

Bernoulli equation along a streamline is:

\[
\frac{P}{\gamma} + \frac{V^2}{2g} + Z = \text{const}
\]  

(1.4)

At point 1, the main stream velocity is \(U_\infty\) and the pressure is \(P_{\text{static}}\), so that Bernoulli equation Bernoulli equation at this point is:

\[
\frac{P_{\text{static}}}{\gamma} + \frac{U_\infty^2}{2g} + Z = \text{const}
\]  

(1.5)

After the flow is stagnated at the open tube, point 2, the velocity at this point reduces to zero, and the pressure increases to \(P_{\text{stagnation}}\). The Bernoulli equation inside the internal tube is:

\[
\frac{P_{\text{stagnation}}}{\gamma} + Z = \text{const}
\]  

(1.6)

For a stream-line along the line 1-2, and for an horizontal Pitot tube, \(Z_1 = Z_2\) and the head in equations (2) is equal to that in (3), resulting in:

\[
\frac{P_{\text{static}}}{\gamma} + \frac{U_\infty^2}{2g} = \frac{P_{\text{stagnation}}}{\gamma}
\]  

(1.7)

Under the assumption that static pressure, \(P_{\text{static}}\), in point 3 is very close to \(P_{\text{static}}\) of point 1, the expression for the main stream velocity is:

\[
U_\infty^2 = \frac{2g}{\gamma} \left[ P_{\text{stagnation}} - P_{\text{static}} \right]
\]  

(1.8)

where the pressures are sensed at point 4 and 5. The equation can be simplified by substituting,

\[
\Delta P = P_{\text{stagnation}} - P_{\text{static}}
\]  

(1.9)

and the equation for fluid density is:
\[ \rho = \frac{\gamma}{g} \] (1.10)

Finally, by substituting Eqs. (1.4 and 1.5) into (1.6), the equation for the velocity gets the simplified form:

\[ U_\infty^2 = 2 \Delta P / \rho \] (1.11)

V. PROCEDURE

1. For one wind-tunnel speed, use the Pitot tube to measure the static and stagnation pressure, and determine the air velocity distribution across the working section of the wind tunnel (between the cylinder and wind tunnel wall). Use small increments of 1 mm near the cylinder for the first 5-mm, and increments of 5 mm for the next 25 mm, and for the rest; increments of 25mm, until you move the Pitot tube all the way across the working section. And reach the wind-tunnel wall. The measured velocity is to be compared to the theoretical velocity distribution in Eq. 4, for \( \phi = 90 \) Deg. The ambient room temperature and pressure as well as relative humidity are to be recorded, and used later to determine the air density and the kinematic viscosity.

2. For each main stream velocity, record the velocity indicated by the wind-tunnel air-speed indicator and compare with one measurement of the Pitot tube.

3. Measure the pressure distribution around the cylinder (0 to 360 Deg.) for at least 4 different air speeds. Do not exceed 90 miles per hour. Determine for each velocity, the location of the separation point.

4. Use ear protectors for this experiment

VI. ANALYSIS

VI.1. Theoretical pressure distribution over a cylinder.

The potential theory offers a solution for the velocity distribution of a fluid past a cylinder in polar coordinate system. The tangential and radial and components are described by Eqs. 4 and 5 respectively,

\[ V_\phi = U_\infty (1 + \frac{R^2}{r^2}) \sin \phi \] (1.12)

\[ V_r = U_\infty (1 - \frac{R^2}{r^2}) \cos \phi \] (1.13)

Since these equations are for potential flow, they apply only to the field outside the boundary layer. The velocity near the wall of the cylinder is, in fact, outside the boundary layer. Since the boundary layer is very thin compared to the radius of the cylinder, \( R \), the velocity along the stream line, near the cylinder wall is calculated from equation (1.12) and (1.14), at \( r = R \).
\[ V_{\phi \ (at\ r=R)} = 2 \ U_{\infty} \ \sin \phi \]  
\[ V_{r \ (at\ r=R)} = 0 \]  

Equation (1.15) describes the radial velocity around the cylinder, in the normal direction to the cylinder. Since the flow does not pass through the cylinder, the radial velocity \( V_{r \ (at\ r=R)} = 0 \).

Equation (1.14) describes the tangential velocity around the cylinder, near the wall. It is interesting to note that the velocity at the wall, at \( \phi = 90 \text{ Deg.} \), is twice the magnitude of the main stream velocity, \( U_{\infty} \).

Using Bernoulli's equation along the streamline adjacent to the cylinder wall, and Eq. (1.14), derive the expression for the pressure distribution around the cylinder. Assume that the cylinder is vertical \( (z = \text{cons.}) \) and that the pressure in the front stagnation point, where \( \phi = 0 \), is zero; \( P_0 = P_{(\phi = 0)} = 0 \). In the report, you are expected to present this derivation, and to compare this analytical pressure distribution with the measured pressure distribution around the cylinder.

### VI.2. Theoretical shear stress distribution around a cylinder

The theoretical shear stress near the wall of the cylinder (i.e. skin friction) has been solved by the boundary layer theory. This solution required a computer program, and the result is represented, in dimensionless form, in Fig. 1-3. The dimensionless shear stress, \( \tau_0 \), at the wall is:

\[ \bar{\tau}_o = \sqrt{2} \ \frac{\tau_o}{\rho U_{\infty}^2} Re^{\frac{\nu}{U_{\infty}}} \]  

![Graph showing theoretical shear stress distribution around a cylinder](image)

**Fig. 1-3: Theoretical shear stress distribution around a cylinder**
The theoretical shear stress at the wall, \( \tau _{0} \), can be determined from Fig. 1-3, if the magnitude of all the constants is known.

**VI.3 Numerical Integration of the drag force components**

The directions of the pressure as well as shear stress vary around the cylinder. Therefore, it is necessary to integrate the components along the flow direction. Since the direction of the pressure varies around the cylinder, in order to allow summation, the elementary force, \( dF \), is divided into components \( dF_x \) and \( dF_y \), each of which can be integrated around the bearing. The elementary force, \( (dF = P \cdot dA) \), per unit cylinder width is obtained as:

\[
dF = P \cdot R \cdot d\phi
\]  
(1.17)

The pressure is in the normal direction to the journal surface, and \( dF \) is a radial vector directed towards the center, as shown in Fig. 1-1. The two components of the force are in the \( x \) and \( y \) directions. The elementary load components are:

\[
dF_x = -P R \cos \theta d\theta
\]  
(1.18)

\[
dF_y = P R \sin \theta d\theta
\]  
(1.19)

The hydrodynamic drag force induced by the pressure, form drag force, is:

\[
F_x = -2L \cdot R \int_{0}^{\pi} P \cos \phi d\phi
\]  
(1.20)

and the hydrodynamic drag force induced by the shear stress, skin friction force, is:

\[
F_x = 2L \cdot R \int_{0}^{\pi} \tau \sin \phi d\phi
\]  
(1.21)

Integration of the shear stress component, in the \( x \)-direction, is performed according to Eq. (13) The integration is in the range between \( \phi = 0 \) and the separation point (the shear stress is zero after the separation point). It yields the part of the skin friction force, which is induced by shear stresses. The shear stresses can be deducted from the dimensionless shear stress in Fig. 1-3.

**Note:** You do not have the functions, because your results are numerical. You must convert Eqs. (1.20 and 1.13) into numerical integration. Use one of the numerical integration methods that are available in the literature.
VI. Report

The following graphs should be included in the report:

1. For one main stream velocity, plot and compare the measured and analytical velocity across the working section of the wind tunnel. Show the distribution from the cylinder wall, \( r = R \), to the wall of the wind tunnel. The analytical distribution is according to Eq. (4) for \( \phi = 90 \) Deg., while the measure distribution is done by the Pitot tube.

2. For each of the four different speeds, plot and compare the measured and theoretical dimensionless pressure, according to Eq. (14), versus the cylinder angle, \( \phi \), measured from the forward stagnation point.

\[
\bar{P} = \frac{P - P_0}{\frac{1}{2} \rho U_\infty^2}
\]

Here \( P \) and \( U_\infty \) are the variable pressure and the main stream velocity in the working section of the wind tunnel, respectively. The pressure, \( P_0 \), is at the front stagnation point where \( \phi = 0 \).

Important: the graphs should be in the range of \( \phi = 0 \) to 180 Deg., however, the measured pressure at each point, should be taken as the average of the two sides of the cylinder (use the measured results \( \phi = 0 \) to 360 Deg. for averaging). On the same coordinate system, plot the dimensionless theoretical pressure distribution and compare with the measured curve. Mark the point of fluid separation.

3. For each measurement of the pressure distribution around the cylinder, calculate by numerical integration the two components of the drag force; form drag and skin friction. The two components of drag should be calculated by integrating the shear stress and pressure distribution from the forward stagnation point to the back stagnation point. For the skin friction, use the theoretical shear distribution in Fig. 2. Convert from dimensionless to regular shear stress, and note that the shear stress is zero after the point of fluid separation. Show how the ratio of the two parts of the drag varies with Reynolds number. Plot and compare the two components of the drag as a function of Reynolds number. Use the same coordinates for comparison of the magnitude of the two force components (in Newton) versus the Reynolds number. One complete sample of the numerical integration, for the form drag and skin friction should be included in the sample calculations.

4. Discuss the comparison of your test results with the theoretical results. Also, compare with published experimental results. In the full report, you are expected to compare to wide range of experimental results, including other practical shapes, such as cars and airfoils.

5. Discuss the trend the force components with Reynolds number. Also, indicate in which direction the separation point moves with increasing velocity, and explain.
6. **Preliminary Report:** will include two curves, for one air velocity, comparing the measured and analytical pressure distributions on one coordinate system. Also, complete sample calculations of numerical integration of the form drag for the measured and theoretical pressure distribution. For the theoretical pressure distribution, assume a constant pressure after the point of fluid separation (equal to the pressure at that point). All preliminary report text and graphs should be hand written by each student. Each student should do the calculations as his own personal effort. The two curves of the measured and calculated pressure distribution in the preliminary report are not normalized. It is the actual pressure, unlike the dimensionless pressure in the final report.

**Example Problem**

In a wind tunnel experiment of flow past a cylinder, a Pitot tube has been used to measure the mainstream velocity and the measured data is:

\[ P_{\text{stagnation}} = -0.030\ [\text{inch } H_2O], \quad P_{\text{static}} = -1.570\ [\text{inch } H_2O] \]

The diameter of the cylinder is 3 inch and the measured pressure distribution at \( \phi = 90^\circ \) are:

\( P \) (measured) = -2.820 [inch. H\(_2\)O]

**Calculate:**

a. main-stream velocity,

b. theoretical pressure at the wall of the cylinder at \( \phi = 90^\circ \) (in Pascal units and dimensionless pressure)

c. actual pressure at the wall of the cylinder at \( \phi = 90^\circ \) (in Pascal units and dimensionless pressure)

d. form drag,

e. and skin-friction drag.

**Solution:**

1) Calculation of the main stream velocity \( U_\infty \)

\[ U_\infty^2 = \frac{2\Delta P}{\rho} \]

\[ \Delta P = P_{\text{stagnation}} - P_{\text{static}} \]

\[ P_{\text{stagnation}} = -0.030\ [\text{inch } H_2O], \quad P_{\text{static}} = -1.570\ [\text{inch } H_2O] \]

\[ \Delta P = [-0.030 + 1.570] = 1.540\ [\text{inch } H_2O] \]

Convert pressure difference from inches of water to SI units (Pascal):
P = 1.540 (inch H₂O)*246 = 378.8 Pa

The density of the air can be determined from a chart:

\[ \rho = 1.16 \text{ kg/m}^3 \]

The mainstream velocity:

\[ U_\infty = \sqrt{\frac{2 \times 378.8 \text{ Pa}}{1.16 \text{ kg/m}^3}} = 25.55 \text{ m/s} \]

b) Theoretical pressure at \( \phi = 90^\circ \)

The equation for the pressure distribution around the cylinder is:

\[ P = -2 \rho U_\infty^2 \sin^2 \phi \]

\[ P = -1.16 \times 2 \times (25.55)^2 \times \sin^2 90 \]

\[ P = -1514.50 \text{ Pa} \]

and the dimensionless theoretical pressure is:

\[ P_{\text{theoretical}} = \frac{P - P_0}{\frac{1}{2} \rho U_\infty^2} = \frac{-1514.50 - 0}{0.5 \times 1.16 \times (25.55)^2} = \frac{-1514.50}{378.62} \]

\[ P_{\text{theoretical}} = -4.0 \]

c) Measured pressure at \( \phi = 90^\circ \)

\[ P_{\text{measured}} = -2.820 \text{ [inch. H₂O]} \]

Converting to SI units: \(-2.820 \times 246 = -693.7 \text{ Pa}\)

and the dimensionless experimental pressure is:
\[ P_{\text{experimental}} = \frac{P - P_0}{\frac{1}{2} \rho U_w^2} = \frac{-693.72 - 0}{0.5 \times 1.16 \times (25.55)^2} \]

\[ P_{\text{experimental}} = -1.832 \]

**Calculation of shear and form drag:**

\[ F_p = 2RL \int_0^\pi p \cos \phi \, d\phi \]

\[ R = 0.0379 \, m \]

\[ L = 0.250 \, m \]

\[ F_p = 0.0185 \times \frac{10 \times \pi}{180} \left[ P_5 \cos 5 + P_{15} \cos 15 + \ldots + P_{175} \cos 175 \right] \]

**Skin friction force:**

\[ F_\tau = 2RL \int_0^\pi \tau \sin \phi \, d\phi \]

\[ F_\tau = \Delta \phi 2RL \sum \tau_i \sin \phi_i \]

\[ F_\tau = \frac{10\pi}{180} 0.0185 \left[ \tau_5 \sin 5 + \tau_{15} \sin 15 + \ldots + \tau_{95} \sin 95 \right] \]

The values of the shear stresses around the cylinder can be determined from the curve. The dimensionless shear is converted to shear \([Pa = \text{Newton} / m^2]\).