1 Chapter 2.2: Graphs of Equations

The graph of an equation in two variables, such as x and y, is the set of all ordered pairs (x, y) in the coordinate plane that sitisfy the equation.

Strategies for Graphing

1. Plotting Points
   - Choose values for the independent variable and calculate corresponding values of the dependent variable
2. Using Transformations
   - Find parent graph
   - Identify shifts and apply them in the proper order
3. Find intercepts to guide graph
   - A x-intercept is the value a where $(a, 0)$ is a point on the graph. A y-intercept is the value b where $(0, b)$ is a point on the graph.
     - Find intercepts by setting the opposite coordinate to zero and solving for the only remaining variable
4. Use Symmetries
   - A graph has symmetry if one portion of the graph is a mirror image of another portion
   - A graph is symmetric with respect to the y-axis if for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph
     - If replacing $x \to -x$ results in an equivalent equation, then it is symmetric with respect to the y-axis
   - A graph is symmetric with respect to the x-axis if for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph
     - If replacing $y \to -y$ results in an equivalent equation, then it is symmetric with respect to the x-axis
   - A graph is symmetric with respect to the origin if for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph
     - If replacing $(x, y) \to (-x, -y)$ results in an equivalent equation, then it is symmetric with respect to the origin

Circles

A circle is a set of points in a Cartesian coordinate plane that are at a fixed distance r from a specified point (h, k). The fixed distance r is called the radius of the circle and specified point (h, k) is called the center of the circle.

From the definition, let $C = (h, k)$ be the center and $P = (x, y)$ be a point on the circle. Then

$$\begin{aligned} d(P, C) &= r, \\ d(P, C)^2 &= r^2, \\ (x-h)^2 + (y-k)^2 &= r^2 \end{aligned}$$

The Standard Form for the Equation of a Circle with center (h, k) and radius r is

$$(x-h)^2 + (y-k)^2 = r^2 \tag{1}$$

The General Form for the Equation of a Circle is

$$x^2 + y^2 + ax + by + c = 0 \tag{2}$$

Example 2.2.1: Find the center and radius of the circle with equation $x^2 + y^2 + 4x - 6y - 12 = 0$.
Solution: We need to complete the square in both x and y:

$$\begin{aligned} x^2 + y^2 + 4x - 6y - 12 &= 0, \\ (x^2 + 4x + \_\_\_) + (y^2 - 6y + \_\_\_) &= 12 + \_\_\_ + \_\_\_, \\ (x^2 + 4x + 4) + (y^2 - 6y + 9) &= 12 + 4 + 9, \\ (x+2)^2 + (y-3)^2 &= 25 \end{aligned}$$

Hence we can now clearly see that the center of the circle is $(-2, 3)$ and the radius of the circle is 5.
Example 2.2.2: Find the standard form equation of the circle with center $(3, -4)$ that passes through the point $(-1, 5)$.
Solution: We need to find the radius of the circle which we can do by calculating the distance between the center and the point on the circle given to us. That is

$$d(C, P) = \sqrt{(3+1)^2 + (-4-5)^2} = \sqrt{97}$$

Hence we have

$$(x-3)^2 + (y+4)^2 = 97$$