1 Chapter 5.5: Graphs of Other Trigonometric Functions

Tangent Function

- Recall that if $P = (x, y)$ is a point on the terminal side of an angle θ in standard position, then $\tan\theta = y/x$
- Because the origin $(0,0)$ is also on the terminal side, the slope of that line is slope $= m = \frac{y-0}{x-0} = \frac{y}{x} = \tan\theta$

Example 5.5.1: A line l makes an angle $\theta = 60°$ with the positive x-axis and passes through the point $P = (7,5)$. Find the equation of l in slope-intercept form.
Solution: We can calculate that slope $= m = \tan 60° = \sqrt{3}$. Using this and the point P, the equation for the line l is

$$y - y_1 = m(x - x_1) \implies y - 5 = \sqrt{3}(x - 7)$$

Domain and Range of the Tangent Function

- The function $y = \tan x$ is defined everywhere except where $\cos\theta = 0$.
- The tangent of an angle θ in standard position is the slope of the line containing the corresponding terminal side. Because every real number is the sope of some line through the origin, every number is the tangent of some angle θ.
  - Hence, we have that the domain of $\tan x$ is $\{x \neq \frac{\pi}{2} + \pi n\}$ and the range is $(-\infty, \infty)$

Other Properties of the Tangent Function

- Zeros: From $\tan x = \frac{\sin x}{\cos x}$ we see that $\tan x$ has the same zeros as $\sin x$. Hence the zeros are at $x = \pi n$ for any integer n
- Period: Note that $\tan(\theta + \pi) = \frac{-y}{-x} = \frac{y}{x} = \tan\theta$ for all angles θ. Hence, $\tan x$ has period of π.
- Odd: Note that $\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin(x)}{\cos(x)} = -\tan x$. Hence, $\tan x$ is an odd function like $\sin x$.
- Five Key Points: Given by Table **??**. Notice that for tangent we have much less information.

$y = \tan x$
$(-\pi/2, \text{Und})$
$(-\pi/4, -1)$
$(0, 0)$
$(\pi/4, 1)$
$(\pi/2, Und)$

Table 1: Listed are the five key points in the interval $[0, 2\pi]$.

Graphs of the Reciprocal Functions
Let $f(x)$ be the reciprocal of $g(x)$: $f(x) = 1/g(x)$ where $g(x)$ is any trig function.

- Periodicity: If $g(x)$ has period P, then $f(x)$ has period P
- Zeros: If $g(c) = 0$, then $f(c) = \text{Und}$. If $g(d) = \text{Und}$, then $f(d) = 0$ [i.e. x-intercepts to vertical asynptotes and vice-versa]
- Even/Odd: If $g(x)$ is odd, then $f(x)$ is odd. If $g(x)$ is even, then $f(x)$ is even.
- Special Values: If $g(x_1) = 1$, then $f(x_1) = 1$. If $g(x_1) = -1$, then $f(x_1) = -1$
- Sign: If $g(x) > 0$ on the interval (a, b), then $f(x) > 0$ on the interval (a, b). If $g(x) < 0$ on the interval (a, b), then $f(x) < 0$ on the interval (a, b).
- Increasing/Decreasing: If $g(x)$ is increasing on the interval (a, b), then $f(x)$ is decreasing over that same interval. If $g(x)$ is decreasing on the interval (a, b), then $f(x)$ is increasing over that same interval.
- Magnitude: If $|g(x)|$ is small, then $|f(x)|$ is large. If $|g(x)|$ is large, then $|f(x)|$ is small.