1 Chapter 7.3: Areas of Polygons Using Trigonometry

Area Formulas

Recall from past geometry classes that we have the area and perimeter formulas given in Table 1 as well as the volume and surface area formulas given in Table 2

	Square	Rectangle	Triangle	Trapezoid	Circle
Area	$= s^2$	$= lw$	$= \frac{1}{2}bh$	$= \frac{1}{2}h(a+b)$	$= \pi r^2$
Perimeter	$= 4s$	$= 2l + 2w$	$= a+b+c$	$= a+b+c+d$	$= 2\pi r$

Table 1: Area and Perimeter formulas for various generic polygons and shapes.

	Cube	Rectangular Prism	Sphere	Right Circular Cylinder	Right Circular Cone
Volume	$= s^3$	$= lwh$	$= \frac{4}{3}\pi r^3$	$= \pi r^2 h$	$= \frac{1}{3}\pi r^2 h$
Surface Area	$= 6s^2$	$= 2(lw + wh + hl)$	$= 4\pi r^2$	$= 2\pi rh + 2\pi r^2$	$= \pi r\sqrt{h^2+r^2} + \pi r^2$

Table 2: Volume and Surface Area formulas for various generic polygons and shapes.

Example 7.3.1: One side of a rectangular plot is 48 feet and its diagonal is 50 feet. Find its area and perimeter. Solution: We have a right triangle with hypotenuse of 50 and one leg of 48. We can thus use the Pythagorean Theorem to calculate the other leg of the right triangle is 14 feet. Thus we want to find the area and perimeter of a rectangle with side lengths of 14 and 48 feet. Using the formulas in the table we find $A = 672$ feet^2 and $P = 124$ feet.

Heron's Formula for SSS Triangles

The area A of an SSS triangle with side lengths a, b, c is

$$A = \sqrt{s(s-a)(s-b)(s-c)} \tag{1}$$

where $s = \frac{1}{2}(a+b+c)$ is the semiperimeter.

- Note that this can be derived from the Law of Cosines.

Polygons

- A closed plane figure bounded by at least three line segments (sides) is called a polygon
- A point where two adjacent sides of a polygon meet is the vertex of a polygon
- An angle formed inside the polygon by two adjacent sides is an interior angle
- A line joining any two non-consecutive vertices is a diagonal of the polygon
- A polygon with all sides of equal length and all interior angles of equal measure is called regular

Facts: For a polygon with n sides:

1. The number of diagonals in a polygon is $\frac{1}{2}n(n-3)$
2. The number of triangles formed by drawing all the diagonals from one vertex is $n-2$
3. The sum of the interior angles is $180(n-2)^\circ$