1 Chapter 8.1: Systems of Linear Equations in Two Variables

- A system of equations is a set of equations with the same variables
  - If each equation is linear, then it is a linear system of equations
  - If at least one equation is nonlinear, then it is a nonlinear system of equations
- A solution of a system of equations in two variables x and y is an ordered pair of numbers (a, b) such that when $x = a$ and $y = b$, the resulting equations are true
- The solution set of a system of equations is the set of all solutions of the system

Example 8.1.1: Use a graph to solve the system:

$$x + y = 4,$$
$$3x - y = 0$$

Solution: A system of equations can be solved graphically where the solution is given by the set of points of intersection. The system of equations we have is linear which means we can graph these lines by finding the slopes and $y - intercepts$. This is shown in Figure 1. The solution is $(1, 3)$.

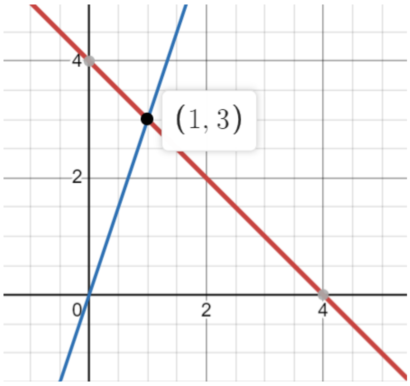

Figure 1: The solution to the linear system of equations in Example 8.1.1 is the set of points of intersection, $(1, 3)$

Example 8.1.2: Use substitution to solve the system:

$$x - y = 5,$$
$$2x + y = 7$$

Solution: The first equation can be rearranged so that $x = 5 + y$. Then substitute this into the second equation to find

$$\begin{aligned} 2x + y &= 7, \\ 2(5 + y) + y &= 7, \\ 10 + 3y &= 7, \\ 3y &= -3, \\ y &= -1 \end{aligned}$$

We then substitute this value of y into our equation for x to find that $x = 5 + (-1) = 4$. Hence the solution is $(4, -1)$.

Example 8.1.3: Solve the system:

$$x - 3y = 1,$$
$$-2x + 6y = 3$$

Solution: The first equation can be rearranged to find that $x = 3y + 1$. Then substitute this into the second equation to find

$$\begin{aligned} -2x + 6y &= 3, \\ -2(3y + 1) + 6y &= 3, \\ -2 &= 3 \end{aligned}$$

which is a false statement. Hence there are no solutions to this system of equations.
Example 8.1.4: Solve the system:

$$\begin{aligned} -2x + y &= -3, \\ 4x - 2y &= 6 \end{aligned}$$

Solution: The first equation can be rearranged to find that $y = 2x - 3$. Then substitute this into the second equation to find

$$\begin{aligned} 4x - 2y &= -3, \\ 4x - 2(2x - 3) &= -3, \\ 4x - 4x + 6 &= 6, \\ 6 &= 6 \end{aligned}$$

which is a true statement. Hence there are infinitely many solutions to this system of equations all given by the set of points $\{(n, 2n - 3) : n \text{ real number}\}$.