

1 Chapter 8.4: Systems of Nonlinear Equations

In this section we discuss nonlinear equations. All of the techniques we have used to solve linear systems can be applied to nonlinear systems.

Example 8.4.1: Solve the system

$$\begin{aligned}x^2 + y &= 2, \\2x + y &= 3\end{aligned}$$

Solution: We will use substitution to solve this system. Note that from the first equation we can find $y = 2 - x^2$. Substituting this into the second equation, we find:

$$\begin{aligned}2x + (2 - x^2) &= 2, \\x^2 + 2x + 1 &= 0, \\(x - 1)^2 &= 0 \implies x = 1 \implies y = 2 - 1 = 1\end{aligned}$$

Hence the solution is $(1, 1)$.

Example 8.4.2: Solve the system

$$\begin{aligned}x^2 + 2y^2 &= 34, \\x^2 - y^2 &= 7\end{aligned}$$

Solution: From the second equation, we find that $x^2 = y^2 + 7$. Substituting this into the first equation, we find

$$\begin{aligned}x^2 + 2y^2 &= 34, \\(y^2 + 7) + 2y^2 &= 34, \\3y^2 &= 27, \implies y = \pm 3\end{aligned}$$

We now plug these values into our other equation to find $x^2 = 9 + 7 = 16 \implies x = \pm 4$. Hence the solution is $\{(-4, -3), (-4, 3), (4, -3), (4, 3)\}$.

Example 8.4.3: Danielle bought 240 shares of stock at \$40 per share and paid \$100 in broker's fees. She kept these for three years during which she received a number of additional shares of stock as dividends. After three years, her dividends were worth \$1950 and she sold all of her stock and made a profit of \$7850 (after paying \$100 in initial broker's commission). How many shares of the stock did she receive as dividends? What was the selling price of each share of the stock?

Solution: We start by defining our variables:

$$\text{Let: } x = \text{the number of shares received as dividends, } p = \text{selling price per share}$$

We are told that she made \$1950 from the dividends and from this we have

$$xp = 1950$$

Further, we know that revenue comes from two streams: the sale of her initial shares and the sale of the shares she received from her dividends. We also know that her costs were the initial cost of the stock when bought it and the one-time broker's fees. Therefore, our profit equation can be determined as:

$$\begin{aligned}\text{Revenue} &= (240 + x)p, \\ \text{Cost} &= (240)(40) + 100 = 9700, \\ \text{Profit} &= \text{Revenue} - \text{Cost}, \\ 7850 &= (240 + x)p - 9700 \implies 240p + xp = 17550\end{aligned}$$

So the nonlinear system of equations we need to solve is

$$\begin{aligned}xp &= 1950, \\17550 &= 240p + xp\end{aligned}$$