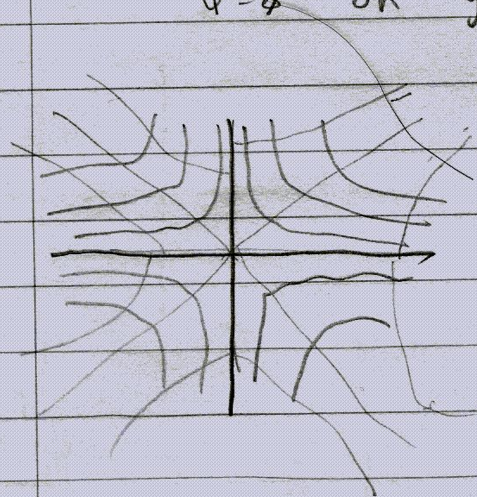
1. W is analytic.

$$W = \frac{A}{2}\left\{(x^2 - y^2) + 2ixy\right\}$$

$$\psi = Axy \,, \quad \phi = \frac{A}{2}(x^2 - y^2)$$

$$\psi = \phi \quad \text{on} \quad y = \phi$$



$$W = Az$$

$$\bar{W} = u + iv = A\bar{z}$$

$W = \cos^{-1} z$

2. $z = \cos W = \dfrac{e^{iW} + e^{-iW}}{2}$ Let $\alpha = e^{iW}$

$$z = \frac{\alpha + \alpha^{-1}}{2}$$

$$2\alpha z = \alpha^2 + 1 \qquad \alpha^2 - 2\alpha z + 1 = \phi$$

$$\alpha = \frac{2z \pm \sqrt{\cancel{4}z^2 - 4}}{2}$$

ie $e^{iz} = z \pm \sqrt{z^2 - 1}$ $z = -i \ln\left(z \pm \sqrt{z^2 - 1}\right)$

Can

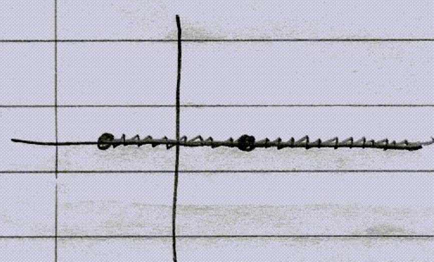
$z \pm \sqrt{z^2-1} \overset{?}{=} \phi$

$z^2 = z^2 - 1$ X

Branch pts: $z = \infty$

$z = \pm 1$

Branch cut



3, (a) $R = \lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right|$

$\frac{a_{n+1}}{a_n} \frac{z^{n+1}}{z^n} < 1$

$z < \underbrace{\frac{a_n}{a_{n+1}}}_{R}$

$= \lim_{n\to\infty} \left| \frac{\frac{n+1}{(3i)^n}}{\frac{n+2}{(3i)^{n+1}}} \right| = \left| \frac{3i\,(n+1)}{n+2} \right|$

$= 3$

(b) $\lim_{n\to\infty} \left(\frac{n+i}{n-2i} \right) = 1$

Hence $R = 1$

(c) Let $\mathfrak{z} = (z - 2 + i)^2$

$$\sum_{0}^{\infty} \frac{1}{n 2^n} \mathfrak{z}^n$$

$$\lim_{n \to \infty} \left| \frac{a_n}{a_{n+1}} \right| = \left| \frac{\frac{1}{n 2^n}}{\frac{1}{(n+1) 2^{n+1}}} \right| = 2(n+1)$$

$\therefore$ Convergence for

$|z - 2 + i|^2 < 2$,

ie $\boxed{|z - 2 + i| < \sqrt{2}}$

4a) $$\int_c \frac{1}{(z-2)(z+1)} dz$$

$$\frac{A}{(z-2)} + \frac{B}{(z+1)} = \frac{1}{(z-2)(z+1)}$$

$$= \int_c \frac{\frac{1}{3}}{(z-2)} + \frac{-\frac{1}{3}}{z+1} dz$$

$A(z+1) + B(z-2) = 1$

$-3B = 1 \quad B = -\frac{1}{3}$

$A = 1/3$

$$= 2\pi i \left(\frac{1}{3} - 1/3 \right) = \phi$$

b) ϕ since $\frac{\sin z}{z}$ analytic

c) $\oint \frac{e^z}{z(1-z)} dz$

$= \oint e^z \left(\frac{1}{z} - \frac{1}{z-1} \right) dz$

$= 2\pi i (1 - e^1)$

d) $\int z^{1/2} dz = 3^{1/2} \int_0^{2\pi} e^{i\theta/2} \, i3e^{i\theta} d\theta$

take pos. branch

$= i\, 3^{3/2} \int_0^{2\pi} e^{i3\theta/2} \, d\theta$

$= \frac{i\, 3^{3/2}}{3i/2} e^{i3\theta/2} \Big|_0^{2\pi}$

$= 2 \cdot 3^{1/2} (-1-1)$

$= -4 \cdot 3^{1/2}$

e) $\oint e^{1/z} dz \qquad t = 1/z \qquad dt = -\frac{1}{z^2} dz$

$= \oint -\frac{e^t}{t^2} dt = -\oint \frac{(1+t+\ldots)}{t^2} = +2\pi i$

5. $z = Re^{i\theta}$

$$\left| \int_0^{\pi/3} \frac{iRe^{i\theta}}{1+R^3e^{3i\theta}} d\theta \right| \le$$

$$\int_0^{\pi/3} \frac{R}{|1-R^3|} d\theta = \frac{\pi/3\, R}{|1-R^3|}$$

and $\lim_{R\to\infty} \frac{\pi/3\, R}{|1-R^3|} = \emptyset.$

6. a) On the unit circle $z = e^{i\theta}$

On $z = e^{i\theta}$

$$|f| = \frac{|e^{2i\theta}|}{|2+e^{2i\theta}|} = \frac{1}{\sqrt{5+2(e^{2i\theta}+e^{-2i\theta})}}$$

$(2+e^{2i\theta})(2+e^{-2i\theta})$

$$= \frac{1}{\sqrt{5+4\cos 2\theta}}$$

This has a max when

$\cos 2\theta = -1$ ie $\theta = \pi/2$

$$\boxed{|f(\theta=\pi/2)| = 1.}$$