Dynamic condensation blocking in cryogenic refueling

V. V. Osipov[1] and C. B. Muratov[2,a)]
[1]*Mission Critical Technologies, Inc., 2041 Rosecrans Avenue, Suite 225, El Segundo, California 90245, USA and Intelligent Systems Division, D&SH Branch, NASA Ames Research Center, MS 269-1, Moffett Field, California 94035, USA*
[2]*Department of Mathematical Sciences, New Jersey Institute of Technology, Newark, New Jersey 07102, USA*

(Received 22 August 2008; accepted 22 October 2008; published online 5 December 2008)

We demonstrate that a negative feedback between vapor pressure and condensation rate may be established in two-phase systems during vapor compression with rates of practical importance. As a result, dynamic condensation blocking occurs. The effect is studied numerically in the case of filling a no-vent insulated tank by liquid hydrogen. It is shown that the filling dynamics quite sensitively depends on the filling rate, and for sufficiently fast filling rates consist of a fast stage dominated by gas compression and a slow stage governed by heat conduction in the liquid. © *2008 American Institute of Physics*. [DOI: 10.1063/1.3025674]

Filling fuel tanks of liquid and hybrid rockets with liquefied gases, primarily liquid hydrogen and oxygen, is a complicated and dangerous procedure. At the same time, similar issues also arise in many other cryogenic applications, in particular, in the studies of feasibility of hydrogen-fueled cars. Cryogenic filling is a dynamical process occurring on the time scales specified by the engineering requirements of a particular device.[1–3] Hence, understanding the dynamics of the filling process under realistic conditions is a problem of great importance.

In this letter, we analyze the effect of *dynamic condensation blocking* due to compensation of evaporation and condensation fluxes at the liquid-vapor interface which crucially determines the dynamics of the filling process. This effect results in a strong negative feedback which suppresses condensation of the gas and maintains highly nonequilibrium conditions in the gas phase on the time scale of filling. We numerically examine the consequences of dynamic condensation blocking in the process of filling a no-vent insulated tank.

During condensation, latent heat is released at the liquid surface. As a result, the temperature T_s of the liquid surface increases, leading to an increase in the evaporation rate and a drop in the net condensation flux. The released heat must be transferred into the bulk of the liquid phase, which occurs by heat conduction. If the heat conduction rate is not sufficiently fast, the temperature T_s of the liquid surface will increase until the evaporation rate compensates the condensation rate, effectively stopping condensation. To quantify these processes, let us consider the distribution of temperature in the thermal boundary layer next to the liquid-vapor interface. If x is the distance from a point inside liquid phase to the liquid-vapor interface, the liquid temperature $\Theta(x,t)$ satisfies the boundary-value problem

$$c_L\rho_L\frac{\partial\Theta}{\partial t} = \varkappa_L\frac{\partial^2\Theta}{\partial x^2}, \quad x>0, \tag{1}$$

$$-\varkappa_L\frac{\partial\Theta}{\partial x} = \frac{J_{\mathrm{cd}}q_h}{A}, \quad x=0, \tag{2}$$

where c_L, ρ_L, and $\varkappa_L$ are the heat capacity, mass density, and the heat conductance of the liquid phase, respectively, J_{cd} is the condensation mass flow, q_h is the latent heat of condensation, and A is the area of the liquid-vapor interface (for simplicity, we neglected the small conductive heat transfer between gas and liquid, as well as a possible setup-dependent effect of liquid mixing which may be important for many specific systems). Using the well-known Hertz–Knudsen relation, we can write the net condensation flow as[4–6]

$$J_{\mathrm{cd}} = J(p,T_s) - J(p_s(T_s),T_s), \tag{3}$$

with $J(p,T)=Ap/(2\pi RT)^{1/2}$, where R is the gas constant of the vapor, and $p_s(T_s)$ is the pressure of saturated vapor at temperature T_s. Here for simplicity we assumed that no gases other than the vapor are present in the gas phase, treated vapor as an ideal gas, and neglected the possibility of droplet nucleation in the gas phase. Given the dependences $p=p(t)$ and $T=T(t)$, the function $T_s(t)=\Theta(0,t)$ obtained by solving the boundary-value problem in Eqs. (1) and (2) with initial temperature distribution $\Theta(x,0)=T_L$ satisfies the integral equation

$$\frac{T_s(t)-T_L}{T_L} = \Omega_0^{1/2}\int_0^t \frac{d\tau}{\sqrt{t-\tau}}\left[\sqrt{\frac{T_L}{T_s(\tau)}}\left(\frac{p(\tau)}{p_{sL}} - \frac{p_s[T_s(\tau)]}{p_{sL}}\right)\right], \tag{4}$$

where $p_{sL}=p_s(T_L)$ and $\Omega_0=p_{sL}^2q_h^2/(2\pi^2\varkappa_L c_L\rho_L RT_L^3)$. We note that for typical parameters of liquid hydrogen[7] at $T_L=20$ K ($p_{sL}\simeq 1$ atm, $q_h=4.47\times10^5$ J/kg, $R=4124$ J/kg K, $\varkappa_L=0.0984$ W/m K, $c_L=9450$ J/kg K, $\rho_L=71.1$ kg/m^3) we have $\Omega_0\sim 5\times10^7\mathrm{s}^{-1}$, hence on the realistic time scales of filling this parameter is very large. Therefore, the only way Eq. (4) can be satisfied is when the vapor pressure is very close to the pressure of saturated vapor at temperature T_s, i.e., T_s solves $p=p_s(T_s)$. Importantly, to very good accuracy

a)Electronic mail: muratov@njit.edu.

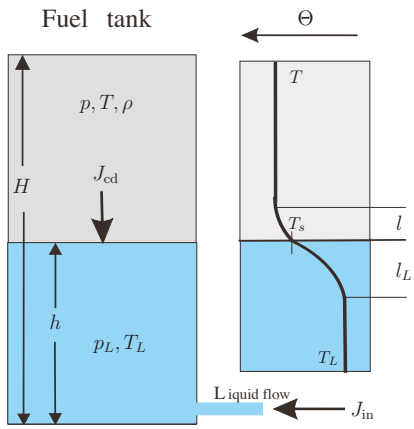

FIG. 1. (Color online) Schematics of the fuel tank and temperature distribution in the liquid and gas phase.

the liquid surface temperature T_s becomes a function of the gas pressure only.

Empirically, it is found that for many gases the saturated vapor pressure follows the dependence $p_s(T)=p_c(T/T_c)^{\lambda}$, where p_c and T_c are the critical pressure and temperature of the gas (e.g., for hydrogen $p_c=1.315$ MPa, $T_c=33.2$ K, and $\lambda=5$).[8] Hence, with very good accuracy the above condition becomes explicitly

$$T_s(p) = T_c(p/p_c)^{1/\lambda}. \tag{5}$$

From Eq. (5), we can now find the time-dependent boundary condition at $x=0$ for Eq. (1), i.e., we have $\Theta(0,t)=T_s[p(t)]$, which uniquely determines the solution of Eq. (1). From this solution one can compute the condensation flux by using Eq. (2), numerically this can be done very efficiently using optimal geometric grid-based finite difference discretizations.[9] Let us note that the net condensation flux J_{cd} obtained in this way will be much less than the individual condensation and evaporation rates in Eq. (3), resulting in condensation blocking.

We now investigate how the phenomenon just described affects the dynamics of filling in a simple closed insulated tank (for earlier studies, based on semiempirical models in the presence of turbulent mixing, see Refs. 10–12). Consider a tank of height H filled with vapor in equilibrium with liquid at temperature T_L, and suppose that at $t=0$ the liquid starts to be pumped into the tank by an external pressure p_0 through a pipe with hydraulic conductance k (Fig. 1). The mass flow of liquid flowing into the tank is then $J_{in}=k(p_0-p)$. Writing the laws of conservation of liquid mass, vapor mass, and gas energy for the liquid height h, vapor density ρ, and vapor temperature T, respectively, after simple algebra we arrive at the following set of equations:

$$\frac{dh}{dt} = \frac{J_{in}+J_{cd}}{\rho_L A}, \tag{6}$$

$$\frac{d\rho}{dt} = \frac{\rho}{A(H-h)}\left(\frac{J_{in}}{\rho_L} - \frac{J_{cd}}{\rho}\right), \tag{7}$$

$$\frac{dT}{dt} = \frac{RT}{c_V A(H-h)}\left(\frac{J_{in}}{\rho_L} - \frac{J_{cd}}{\rho}\right), \tag{8}$$

where c_V is the vapor specific heat at constant volume. Here we took into account that $\rho \ll \rho_L$, and used the ideal gas equation of state $p=\rho RT$ for vapor. To simulate this system, we use the parameters of hydrogen[7] (see above; $c_V=6490$ J/kg K), with typical parameters of the geometry $H=1$ m and $A=0.05$ m^2, respectively, the external pressure $p_0=3$ atm, and $k=1\times10^{-7}$kg/Pa s. The latter is chosen to ensure that filling would occur within 5 min in the absence of condensation blocking, i.e., when the vapor always remains in equilibrium with the liquid.

Figure 2, which shows the result of this simulation, dem-

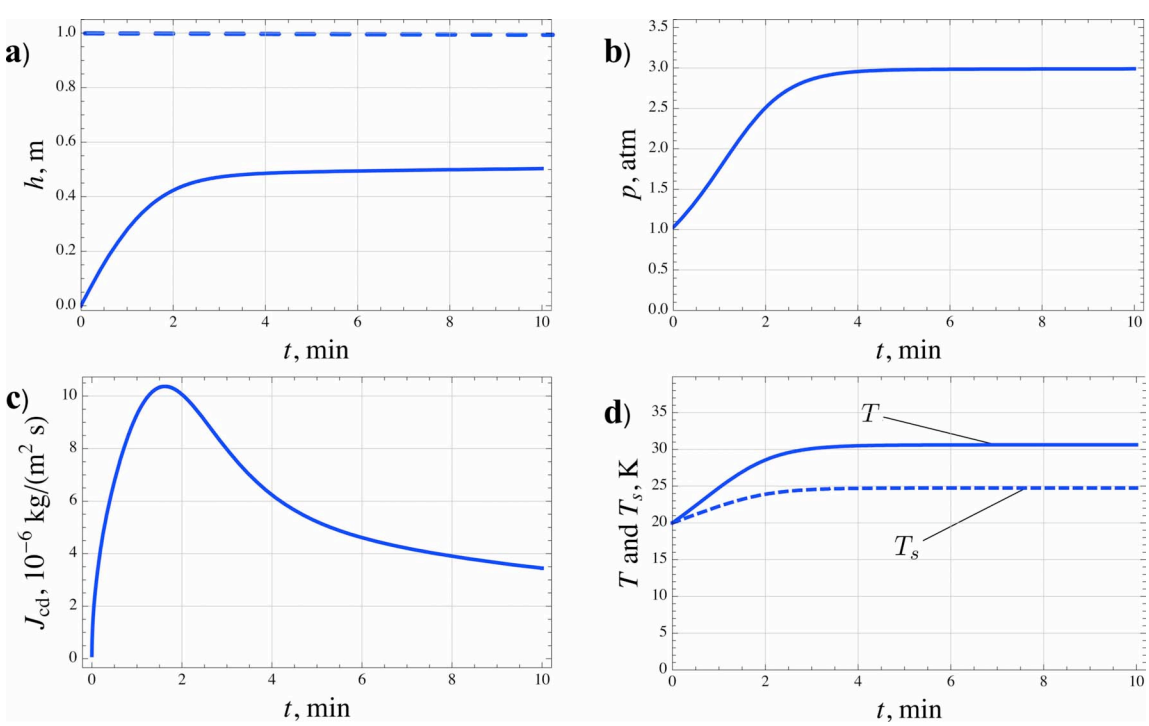

FIG. 2. (Color online) Dynamic condensation blocking during filling of a closed insulated tank. In (a), dashed line corresponds to $h=H$.

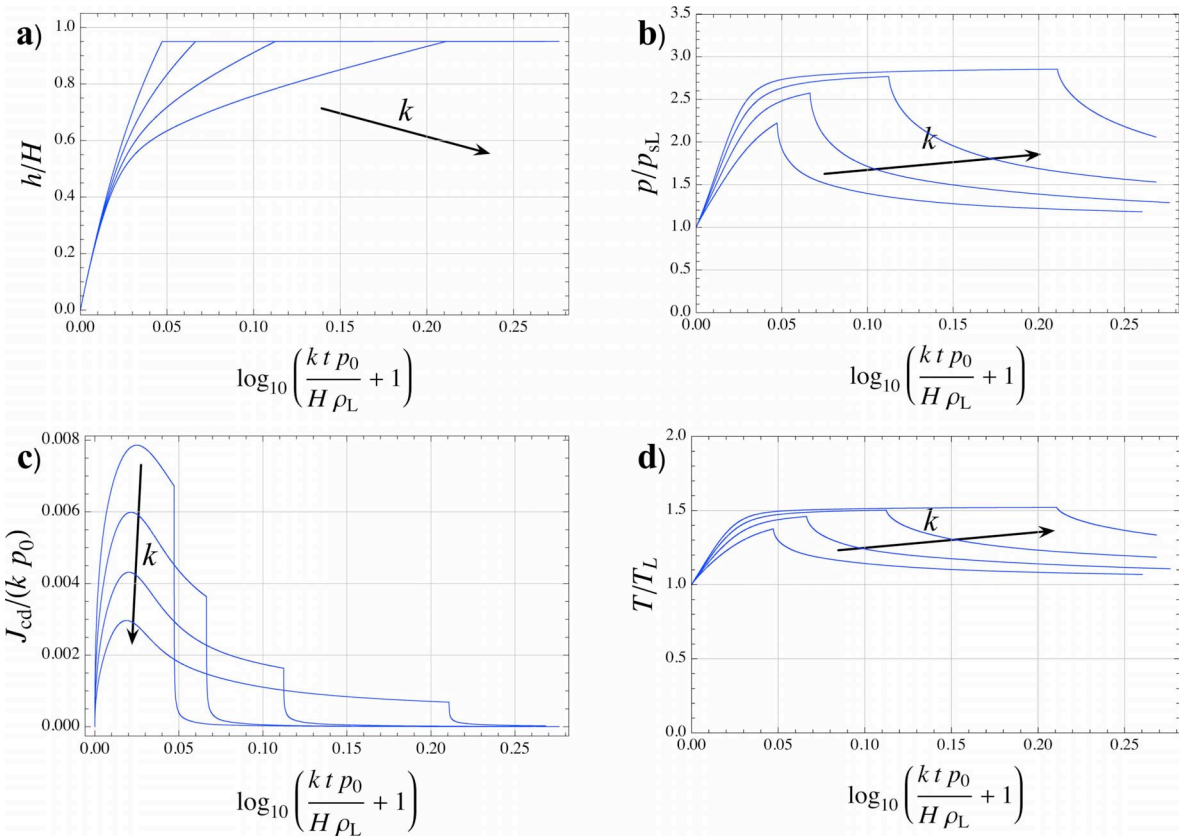

FIG. 3. (Color online) Time traces of the main parameters at different filling rates: $k=6.4\times10^{-11}$, 1.6×10^{-10}, 4×10^{-10}, and 1×10^{-9} kg/Pa s. For these values of k, filling is terminated at about $t=110$, 68, 47, and 40 h respectively.

onstrates that dynamic condensation blocking has a significant effect on the dynamics of filling. Indeed, at first filling proceeds at high rate since the pressure difference p_0-p is large. However, as the volume of the gas decreases, the condensation flux is not sufficient to remove the vapor from the tank, hence the vapor begins to compress. As a result of this compression, the pressure inside the tank begins to build up [Fig. 2(b)], resulting also in an increase in both the vapor temperature T and the liquid surface temperature T_s [Fig. 2(d)]. Nevertheless, the increase in T_s is not sufficient to maintain an increased condensation flux, and so, after reaching a maximum soon after the beginning of filling, the condensation flow J_{cd} begins to decrease [Fig. 2(c)]. All these processes lead to an effective stop of filling after only about 50% of the tank has been filled [Fig. 2(a)]. Our simulations show that one needs another 35 h to fill the tank to 95% capacity in this case.

To further investigate the effect of the filling rate on the dynamics, we performed a series of simulations for different values of k (Fig. 3). As can be seen from Fig. 3, when k is sufficiently large, the dynamics of filling slows down considerably after a fast initial stage. This happens because the vapor pressure inside the tank becomes close to the external pressure p_0, and at this point condensation is necessary to produce a pressure difference between p_0 and p. Figure 3 also shows the relaxation of the main parameters toward equilibrium after the tank is filled to 95% capacity and the inflow of liquid is switched off. Note that this relaxation occurs even in the absence of heat exchange between the liquid and gas. The latter is simple to understand: the gas that remains at the end undergoes a reversible adiabatic process. When the filling rate is decreased, the relative duration of the slow stage decreases, until at sufficiently slow filling rates the process occurs at nearly constant temperature and pressure, expected in the limit of adiabatically slow process.

In conclusion, we demonstrated that dynamic condensation blocking is an important phenomenon that must be taken into account in modeling and design of cryogenic systems, such as the ones involved in the new generation of spacecrafts currently under development.

We are grateful to V. N. Smelyanskiy for suggesting this problem to us. The work of C.B.M. was supported, in part, by NSF via Grant No. DMS-0718027.

[1]C. N. Torre, J. A. Witham, E. A. Dennison, R. C. McCool, and M. W. Rinkerf, J. Spacecr. Rockets **26**, 368 (1987).
[2]D. M. DeFelice and J. C. Aydelott, NASA Technical Memorandum 89921 Technical Report No. AIAA-87-1764, 1987.
[3]P. N. Estey, D. H. Lewis, Jr., and M. Connor, J. Spacecr. Rockets **20**, 49 (1983).
[4]L. D. Koffman, M. S. Plesset, and L. Lees, Phys. Fluids **27**, 876 (1984).
[5]K. Aoki and C. Cercignani, Phys. Fluids **26**, 1163 (1983).
[6]Y.-P. Pao, Phys. Fluids **14**, 306 (1971).
[7]R. McCarty, J. Hord, and H. Roder, *Selected Properties of Hydrogen (Engineering Design Data)*, Natl. Bur. Stand. (U. S.) Monograph No. 168 (U.S. GPO, Washington, D.C., 1981).
[8]J. A. Clark, IAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, 2004 (unpublished), paper No. AIAA 2004-4088.
[9]D. Ingerman, V. Druskin, and L. Knizhnerman, Commun. Pure Appl. Math. **53**, 1039 (2000).
[10]D. A. Vaughan and G. R. Schmidt, J. Spacecr. Rockets **28**, 574 (1991).
[11]M. Hemlick, B. Khoo, J. Brown, and A. Sonin, AIAA Paper No. 88-0559, 1988.
[12]J. Brown, M. Hemlick, and A. Sonin, AIAA Paper No. 89-2846, 1989.