1. Prove the following polynomial is $\Theta(n^3)$.

   \[ P(n) = 8n^3 - 20n^2 + 50n + 100 \]

   (a) Prove $O(n^3)$:

   \[
   \begin{tabular}{|c|c|}
   \hline
   GRADE & /20 \\
   \hline
   1 & /20 \\
   2 & /20 \\
   3 & /20 \\
   4 & /20 \\
   5 & /20 \\
   \hline
   SUM & /100 \\
   \hline
   \end{tabular}
   \]

   (b) Prove $\Omega(n^3)$:
2. Consider the relation \( R = \{(1, 1), (1, 3), (2, 1), (2, 2), (3, 2)\} \).

(a) Does the relation satisfy each of following properties? Explain.
- Reflexive?
- Symmetric?
- Antisymmetric?
- Transitive? (Decide this by directly examining the ordered pairs. Do NOT use matrix multiplication for this part.)
- Partial Order?
- Equivalence Relation?

(b) Show the matrix of this relation. Use matrix multiplication to decide if the relation is transitive. Explain.
3. The following pseudocode computes the sum of an array of $n$ integers, $A[0..n-1]$.

```
int sum (int A[], int n) {
    T = 0;
    for i = 0 to n - 1
        T = T + A[i];
    return T;
}
```

(a) Write a recursive version of this program.

(b) Let $f(n)$ be the number of additions performed by your recursive algorithm. Write a recurrence equation for $f(n)$.

(c) Guess the solution of the recurrence equation, and prove the correctness of it by induction.
4. Consider the following recurrence equation. (Assume $n$ is a power of 2.)

$$T(n) = \begin{cases} 
8T(n/2) + n^2, & n \geq 2 \\
1, & n = 1.
\end{cases}$$

Prove by induction the solution is $T(n) = An^3 + Bn^2$, and find the constants $A, B$. 
5. The following divide-and-conquer algorithm performs some simple computation on an array $A$, where $S$ is the starting index, and $n$ is the number of elements. The initial call is COMPUTE($A, 0, n$). The array size $n$ is assumed an integer power of 2.

(a) Figure out what the program computes. Provide a brief explanation.

```c
int COMPUTE (int A[], int S, int n ) {
    int T1, T2;
    if (n == 1) return A[S];
    T1 = COMPUTE (A, S, n/2);
    T2 = COMPUTE (A, S + n/2, n/2);
    return (T1 + T2)
}
```

(b) Let $f(n)$ be the number of times the addition step ($T1 + T2$) is executed when the call is COMPUTE($A, 0, n$). Write a recurrence equation for $f(n)$.

(c) Find the solution by repeated substitution.