CS 241, Fall 2014, Exam 1 Time: 1.5 Hours Name:_____

Instructor: Xiguo Ma First Last

	GRADE	
1		/20
2		/20
3		/20
4		/20
5		/20
SUM		/100

Basic Rules (Equalities) in Set Algebra	
Associative Laws:	$(X \cup Y) \cup Z = X \cup (Y \cup Z)$
	$(X \cap Y) \cap Z = X \cap (Y \cap Z)$
Commutative Laws:	$X \cup Y = Y \cup X$
	$X \cap Y = Y \cap X$
Distributive Laws:	$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$
	$X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$
De Morgan's Laws:	$\overline{X \cap Y} = \overline{X} \cup \overline{Y}$
	$\overline{X \cup Y} = \overline{X} \cap \overline{Y}$
Complement Laws:	$X \cup \overline{X} = U$
	$X \cap \overline{X} = \phi$
	$\overline{\overline{X}} = X$
Repetition:	$X \cup X = X$
	$X \cap X = X$
0/1 Laws:	$\overline{\phi} = U$
	$\overline{U} = \phi$
Identity:	$X \cup \phi = X$
	$X \cap U = X$
Bound Laws:	$X \cup U = U$
	$X \cap \phi = \phi$

1. Prove each of the following set equality by algebraic method.

   (a)
$$A \cap (\overline{A} \cup B) = A \cap B$$

   (b)
$$(A \cap B) - (A \cap C) = (A \cap B) - C$$

2. (a) Consider the proposition

$$P \to Q$$

- What is the equivalent contrapositive form of this proposition?

- Give the negation of the original proposition using only $\wedge$, $\vee$, $\neg$ operations.

(b) Use a truth-table to show the following propositions are logically equivalent.

- $X \leftrightarrow Y$
- $(X \to Y) \wedge (Y \to X)$
- $(X \wedge Y) \vee (\neg X \wedge \neg Y)$

(c) Determine the truth value of each of the following propositions, where the domain is set of positive integers. Justify your answers.

- $\forall x \exists y,\ (x > y)$

- $\exists x \forall y,\ (x \leq y)$

3. (a) Prove the following set equality is true if and only if sets A and B are disjoint.

$$(A \cup B) - B = A$$

Break the proof in two parts:

- If A and B are disjoint, then the above set equality holds.

- If A and B are not disjoint, then the above set equality does not hold.

(b) Use algebraic method to prove the following set equality.

$$(A \cup B) - B = A - B$$

4. (a) Prove by induction that for all integers $n \geq 1$,

$$f(n) = 8^n - 1$$

is divisible by 7.

(b) Let x and y be real numbers.

i. Use contrapositive method to prove that if x^2 is irrational, then x is irrational.

ii. Prove by contradiction that if $x + y > 50$ then at least one of the two numbers is greater than 25.

5. Let $Ace(S, C)$ be the propositional function (predicate) "student S gets an A in course C." Let the domain for S and C be the set of NJIT students and NJIT courses, respectively.

   (a) Express each of the following propositions in symbolic form.

      i. There are NJIT students with all A's.

      ii. There are NJIT students with no A's.

      iii. Every NJIT student gets some A's.

      iv. Every NJIT class gives some A's.

   (b) Express the negation of each of the above propositions, both in words and in symbolic form.

      i.

      ii.

      iii.

      iv.