1. Consider the relation \( R = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 3)\}\).

   (a) Show the matrix of this relation. By direct observation of the matrix (without matrix multiplication) determine if the relation satisfies the following properties. Provide a reasoning.

   Reflexive?

   Symmetric?

   Antisymmetric?

   Transitive?

   Partial Order?

   Equivalence Relation?

   (b) Now, use matrix multiplication to decide if the relation is transitive. Explain.
2. Prove the following polynomial is $\Theta(n^4)$.

$$P(n) = 5n^4 - 2n^3 - 10n^2 + 50n + 100.$$ 

(a) Prove $O(n^4)$:

(b) Prove $\Omega(n^4)$:
3. The following program computes \((\log_2 n)\), assuming \(n\) is an integer power of 2.

(a) Trace the working of the algorithm for \(n = 16\). (Show the values of \(m\) and \(k\) after each iteration of the while loop.)

```c
int LOG (int n){
int m, k;
m = n; k = 0;
while (m > 1)
    {m = m/2; k = k + 1}
return (k)
}
```

(b) Write a recursive version of this algorithm.

(c) Let \(f(n)\) be the number of division operations performed by your recursive algorithm to compute \((\log_2 n)\). Write a recurrence equation for \(f(n)\). Guess the solution and prove it correct by induction.
4. Consider the following divide-and-conquer recursive algorithm. Parameter $i$ is the starting index of array $A$, and $n$ is the number of elements. The initial call is $\text{COMP}(A, 0, n)$.

Boolean $\text{COMP}$ (int $A[\ ]$, int $i$, int $n$) {
  Boolean $C_1, C_2$;
  1. if ($n == 1$) return FALSE ;
  2. $C_1 = \text{COMP} (A, i, \lfloor n/2 \rfloor);$  
  3. $C_2 = \text{COMP} (A, i + \lfloor n/2 \rfloor, n - \lfloor n/2 \rfloor);$  
  4. return ($C_1 \lor C_2 \lor (A[i] \neq A[i + n - 1]));$
}

(a) Figure out what the algorithm does. When does it return TRUE? Explain.

(b) Let $f(n)$ be the number of key comparisons (in step 4) performed by this algorithm for an array of size $n$. Consider the special case when $n$ is a power of 2. Write a recurrence for $f(n)$. Find the solution of the recurrence by repeated substitution.
5. Find the exact solution of the following linear recurrence: $F_0 = 0, F_1 = 9$, and

$$F_n = 7F_{n-1} - 10F_{n-2}, \quad n \geq 2.$$