1. (a) Prove the following set equality both by a Venn Diagram and by algebraic method.

\[ A \cup (\overline{A} \cap B) = A \cup B \]

(b) Prove the following set equality by algebraic method.

\[ (A \cup B) - (A \cap B) = (A \cap \overline{B}) \cup (\overline{A} \cap B) \]
2. (a) Use a truth-table to show the following propositions are logically equivalent. (Show the detailed step-by-step computations in the table.)
   
   i. \( X \leftrightarrow Y \) (if and only if)
   ii. \( (X \to Y) \land (Y \to X) \)
   iii. \( (X \to Y) \land (\neg X \to \neg Y) \)

(b) Express an equivalent form for the following proposition using only \( \land, \lor, \neg \) operations. Use a truth table to show that they are equivalent.
\( (P \to Q) \)

(c) Find an equivalent form for the following proposition using only \( \land, \lor, \neg \) operations.
\( \neg[\text{if } (x < y) \text{ then } (x^2 < y^2)] \)
3. (a) Determine the true/false value of each of the following propositions, where the domain of $x$ and $y$ is all integers. Explain your reasoning.
   
i. $\forall x \exists y, \ y^2 = x^2$

   ii. $\exists x \forall y, \ y^2 = x^2$

(b) Express the negation of each of the above statements and simplify.
4. (a) Given a rational number $x$ and an irrational number $y$. Prove by contradiction that $x \cdot y$ is irrational.

(b) Suppose the domain of $n$ is positive integers. Express the contrapositive equivalent form of the following proposition. Then prove the proposition is true.

If $n^2$ is not divisible by 4, then $n$ is not divisible by 2.
5. (a) Prove by simple induction that any postage amount of \( n \) cents, where \( n \geq 18 \) may be achieved by using only 7-cent stamps and 4-cent stamps. That is, prove that for every integer \( n \geq 18 \), there exist some non-negative integers \( A \) and \( B \) such that

\[ n = 7A + 4B. \]

(b) Is the same true for all \( n \geq 16 \)? Obviously the base is true for \( n = 16 \), since 16 = 7 \(
\times 0 + 4 \times 4 \). So does the induction step also work for all \( n \geq 16 \)? Explain.