CS 241, Spring 2014, Exam 1 Exam #: _____ Name:_____

Prof. David Nassimi (Time: 1.5 Hours) First Last

1. (a) Prove the following set equality both by a Venn Diagram and by algebraic method.

$$A \cup (\overline{A} \cap B) = A \cup B$$

	GRADE	
1		/20
2		/20
3		/20
4		/20
5		/20
SUM		/100

(b) Prove the following set equality by algebraic method.

$$(A \cup B) - (A \cap B) = (A \cap \overline{B}) \cup (\overline{A} \cap B)$$

2. (a) Use a truth-table to show the following propositions are logically equivalent. (Show the detailed step-by-step computations in the table.)

   i. $X \leftrightarrow Y$ (if and only if)

   ii. $(X \rightarrow Y) \wedge (Y \rightarrow X)$

   iii. $(X \rightarrow Y) \wedge (\neg X \rightarrow \neg Y)$

   (b) Express an equivalent form for the following proposition using only $(\wedge, \vee, \neg)$ operations. Use a truth table to show that they are equivalent.
   $(P \rightarrow Q)$

   (c) Find an equivalent form for the following proposition using only $(\wedge, \vee, \neg)$ operations.
   $\neg$[if $(x < y)$ then $(x^2 < y^2)$]

3. (a) Determine the true/false value of each of the following propositions, where the domain of x and y is all integers. Explain your reasoning.

   i. $\forall x \exists y, \quad y^2 = x^2$

   ii. $\exists x \forall y, \quad y^2 = x^2$

   (b) Express the negation of each of the above statements and simplify.

4. (a) Given a rational number x and an irrational number y. Prove by **contradiction** that $x * y$ is irrational.

(b) Suppose the domain of n is positive integers. Express the contrapositive equivalent form of the following proposition. Then prove the proposition is true.

If n^2 is not divisible by 4, then n is not divisible by 2.

5. (a) Prove by **simple induction** that any postage amount of n cents, where $n \geq 18$ may be achieved by using only 7-cent stamps and 4-cent stamps. That is, prove that for every integer $n \geq 18$, there exist some non-negative integers A and B such that

$$n = 7A + 4B.$$

(b) Is the same true for all $n \geq 16$? Obviously the base is true for $n = 16$, since $16 = 7 * 0 + 4 * 4$. So does the induction step also work for all $n \geq 16$? Explain.