1(a) Use a truth table to find a sum-of-minterms expression for the exclusive-or function $A \oplus B$.

(b) Use algebraic manipulations to convert the following Boolean function to a simplified sum-of-products form.

$$f = (X + Y) \oplus Z$$

(c) Use a truth table to compute the above function $f$ starting with its original form. Then, use a map to find a simplified sum-of-products form.
2. Use formal procedures to design a synchronous 4-state gray-code counter with D type flip-flops ($Y_1, Y_0$). The counter has repeated consecutive counts

$00, 01, 11, 10$.

The counter has no external inputs (except the CLOCK), and it advances its count every clock cycle. Draw the final circuit.
3. A synchronous sequential circuit has two flip-flops $(Y_1, Y_0)$ and an external input $X$. The flip-flops are $T$ type and have the following input equations:

$$T_1 = XY_0, \quad T_0 = X.$$ 

Use formal procedures to analyze the circuit. Derive the state transition table and state diagram. Describe what the circuit does.
4. A sequential circuit (sequence detector) has a serial input $X$ and an output $Z$. Whenever the circuit receives a sequence of 0001 (three consecutive 0 followed by a 1), it produces an output $Z = 1$ at the same time as the fourth input. At all other times, the output $Z = 0$. An example input/output sequence is shown below.

$$\begin{align*}
X: & \quad 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0 \\
Z: & \quad 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \\
\end{align*}$$

Starting with an initial state $A$, obtain a state diagram for this sequence detector. (You don’t need to carry out the rest of the design.)
5. A 3-bit *shifter circuit* is a **combinational circuit** with a 3-bit input \((A_2, A_1, A_0)\) and a 3-bit output \((Z_2, Z_1, Z_0)\). The circuit has two control inputs \((S_1, S_0)\) which specify the operation to be performed as follows:

<table>
<thead>
<tr>
<th>(S_1)</th>
<th>(S_0)</th>
<th>Action</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0</td>
<td></td>
<td><em>Straight</em></td>
<td>If input is 011, output is 011.</td>
</tr>
<tr>
<td>0 1</td>
<td></td>
<td><em>Right Rotate</em></td>
<td>If input is 011, output is 101.</td>
</tr>
<tr>
<td>1 0</td>
<td></td>
<td><em>Left Rotate</em></td>
<td>If input is 011, output is 110.</td>
</tr>
<tr>
<td>1 1</td>
<td></td>
<td>Not used.</td>
<td></td>
</tr>
</tbody>
</table>

(a) Write the Boolean equation for output bit \(Z_0\).

(b) Draw the complete circuit for the 3-bit shifter circuit using a decoder and AND-OR gates.