1a) Convert the following unsigned binary integer

\[ 1000111 \]

to each of the following bases. (Be sure to show your computation.)

1. Octal
2. Hexadecimal
3. Decimal

\[
\begin{array}{|c|c|}
\hline
{\text{GRADE}} & /20 \\
\hline
1 & /20 \\
2 & /20 \\
3 & /20 \\
4 & /20 \\
5 & /20 \\
\hline
{\text{SUM}} & /100 \\
\hline
\end{array}
\]

b) Assuming a 16-bit word size and 2’s complement number system, find the binary representations for the decimal integers: \( A = -32 \) and \( B = +8 \). Then, perform the binary addition \( A + B \).

\( A: \)

\( B: \)

\( A + B: \)
2a) Using algebraic manipulations, convert the following Boolean expression to a simplified sum-of-products form.

$$WX + WY(XZ + Z)$$

2b) Using algebraic manipulations, convert the same Boolean expression to a simplified product-of-sums form.

$$WX + WY(XZ + Z)$$
3. Consider the following Boolean function: $f(X, Y, Z) = (XY) \oplus Z$.

a) Use a truth table to obtain both the sum-of-minterms expression and product-of-maxterms expression for $f$. (Express each expression both in algebraic form and decimal form.)

Truth Table:

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y$</th>
<th>$Z$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Sum-of-minterms:

Product-of-maxterms:

b) Use a map to obtain a simplified sum-of-products expression for $f$

c) Implement the function using only NAND gates.
4. Consider the Boolean function

\[ f(W, X, Y, Z) = \Sigma_m(0, 1, 4, 5, 6, 7, 13, 15) + \Sigma_d(3, 9, 11) \]

a) Use a map to obtain a simplified sum-of-products expression for \( f \).

b) Use a map to obtain a simplified product-of-sums expression for \( f \).

c) Implement the function \( f \) using only NOR gates.
5. A comparator circuit has two 2-bit inputs $A = (A_1, A_0)$ and $B = (B_1, B_0)$. (Note that $A_0$ and $B_0$ are the least significant bits of $A$ and $B$. The inputs $A$ and $B$ are unsigned integers in the range 0 through 3.) The circuit compares $A$ and $B$ and produces the result by three outputs $S$, $E$, and $G$. (One of the three outputs will be 1 and the other two will be 0.)

$S$ (smaller): This output will be 1 when $A < B$.
$E$ (equal): This output will be 1 when $A = B$.
$G$ (greater): This output will be 1 when $A > B$.

a) Use a truth table to show the three functions $S$, $E$, and $G$. (List the variables in the order: $A_1, A_0, B_1, B_0$.)

b) Implement the three functions $S$, $E$, $G$ using a decoder and as few additional gates as possible.