Chapter 2: Boolean Algebra, Karnaugh Map Simplification

1. Use either a truth table or algebraic manipulations to prove each of the following Boolean identities. (The method to be used is indicated for each case.)
   (a) DeMorgan’s theorem for 3 variables: \( X'Y'Z' = X + Y + Z \) (Truth Table method)
   (b) Consensus law: \( XY' + X'Z' + YZ = XY + XZ \) (Truth Table method)
   (c) \((X' + Y')(X' + Y') = XYZ + X'Y'\) (Algebraic method)
   (d) \(XYZ + X'Y + XZ = X\) (Algebraic method)

2. Use algebraic manipulations to obtain a simplified sum-of-products expression for each function:
   (a) \( f = XY + X'Y \)
   (b) \( f = (WX + W'XY)(W'X + X'Y) \)

3. (a) The exclusive-or of two variables \((X \oplus Y)\) has a value of 1 when one of the two variables is one and the other is 0. (That is, when an odd number of the variables are 1.) Use a truth table to obtain the sum-of-minterms and product-of-maxterms expressions for \(X \oplus Y\).
   (b) Use algebraic manipulations to simplify the following expression.
   \((X \oplus Y)(X + Y)\)

4. Use a Karnaugh map to obtain a simplified sum-of-products expression for each function:
   (a) \( f(X, Y, Z) = \sum_m(0, 1, 2, 5, 7) \)
   (b) \( f(X, Y, Z) = \overline{X}Y + YZ + XZ + XYZ \)
   (c) \( f(W, X, Y, Z) = \sum_m(0, 2, 4, 6, 7, 9, 10, 13, 15) \)
   (d) \( f(W, X, Y, Z) = \sum_m(0, 3, 5, 8, 10, 11, 13) + \sum_d(7, 15) \)

5. Use a Karnaugh map to find a simplified product-of-sums form for each function:
   (a) \( f(X, Y, Z) = \sum_m(0, 1, 2, 5, 7) \)
   (b) \( f(X, Y, Z) = \overline{X} \overline{Y} + YZ + XZ + XYZ \)
   (c) \( f(X, Y, Z) = \Pi_M(0, 1, 6, 7) \)
   (d) \( f(W, X, Y, Z) = \sum_m(0, 3, 5, 8, 10, 11, 13) + \sum_d(7, 15) \)