1. Prove the following polynomial is \( \Theta(n^4) \).

\[ P(n) = 5n^4 + 2n^3 - 10n^2 - 50n - 100 \]

(a) Prove \( O(n^4) \):

(b) Prove \( \Omega(n^4) \):
2. Find the exact number of times (in terms of $n$) the innermost statement ($X = X + 1$) is executed in the following code. That is, find the final value of $X$. Then express the total running time in terms of $O(\cdot)$.

```
X = 0;
for i = 1 to n - 3
  for j = 2i + 1 to 3n + 5
    X = X + 1;
```
3. Consider the following divide-and-conquer algorithm (recursive function). Parameter \( i \) is the starting index of the array, and \( n \) is the number of elements. The initial call is \( \text{COMPUTE}(A, 0, n) \).

```c
int COMPUTE (int A[ ], int i, int n) {
    if (n == 1)
        return (A[i] == 0) ? 0 : 1;
    n1 = \lfloor n/2 \rfloor; //Length of first half of array
    n2 = n - n1; //Length of second half of array
    C1 = COMPUTE (A, i, n1);
    C2 = COMPUTE (A, i + n1, n2);
    return (C1 + C2)
}
```

(a) Figure out what the function does. (What does it compute?) Explain briefly.

(b) Let \( f(n) \) be the number of times the arithmetic operation \( C1 + C2 \) is performed by this algorithm. Assume \( n \) is a power of 2. Write a recurrence for \( f(n) \). Guess the solution of the recurrence and prove the correctness by induction.

(c) Now consider the general case where \( n \) is any integer. Write a recurrence for \( f(n) \). Guess the solution again and prove it correct by induction.
4. (a) Consider the following recurrence equation. (Assume \( n \) is a power of 2.)

\[
T(n) = \begin{cases} 
8T(n/2) + n, & n \geq 2 \\
1, & n = 1.
\end{cases}
\]

Prove the solution is \( T(n) = An^3 + Bn \), and find the constants \( A, B \).

(b) Use repeated substitution to find the solution of the following recurrence. (Assume \( n \) is a power of 2.)

\[
T(n) = \begin{cases} 
2T(n/2) + n^2, & n \geq 2 \\
1, & n = 1.
\end{cases}
\]
5. Find the solution of the following linear recurrence.

\[
F_n = \begin{cases} 
0, & n = 0 \\
1, & n = 1 \\
8F_{n-1} - 15F_{n-2}, & n \geq 2 
\end{cases}
\] (1)