1. (a) Prove the following set equality both by a Venn Diagram and by algebraic method.

$$(A \cup B) - B = A - B$$

(b) Prove the following set equality is true if and only if $A$ and $B$ are disjoint.

$$A - B = A$$
2. (a) Use a truth-table to show the following propositions are logically equivalent. (Show the detailed step-by-step computations in the table.)
   i. $A \leftrightarrow B$
   ii. $(A \lor \neg B) \land (B \lor \neg A)$

(b) Let $A$ and $B$ be two sets. Find the negation of the following proposition and simplify.

   $$S : \forall x, [(x \in A) \rightarrow (x \in B)]$$

(c) What does the proposition $S$ state in simple words? Express it in a simpler form using set operations.
3. (a) Determine the true/false value of each of the following propositions, where the domain of \( x \) and \( y \) is non-negative integers. Explain your reasoning.

   i. \( \forall x \exists y, \ (y > x) \)

   ii. \( \exists x \forall y, \ (y > x) \)

(b) Express the negation of each of the above statements and simplify.
4. (a) Given a rational number $x$ and an irrational number $y$. Prove by contradiction that $x - y$ is irrational.

(b) Suppose the domain of $n$ is positive integers. Express the contrapositive equivalent form of the following proposition. Then prove the correctness of it.

If $n^2$ is not divisible by 3, then $n$ is not divisible by 3.
5. (a) Prove by simple induction that any postage amount of \( n \) cents, where \( n \geq 30 \) may be achieved by using only 7-cent stamps and 6-cent stamps. That is, prove that for every integer \( n \geq 30 \), there exist some non-negative integers \( A \) and \( B \) such that

\[
n = 7A + 6B.
\]

(b) Next, use strong induction to prove the same postage problem.