1. (a) Prove that \( P(n) = 5n^3 + 50n^2 - 100n \) is \( \Theta(n^3) \).
   
   i. Prove \( \Omega(n^3) \).

   ii. Prove \( O(n^3) \).

   (b) Consider the following pseudo-code. (The code does not do anything useful except to test you!)

   ```python
   x = 0;
   for J = 1 to n
       for K = J+1 to 3*n
           x = x + 1;
   ```

   Let \( T(n) \) be the total number of times the innermost statement (increment \( x \)) is executed. Derive the EXACT value of \( T(n) \). Then express the result in \( O(\ ) \) form.
2. Use induction to prove that every postage of 13 cents or more can be achieved using only 3-cent stamps and 7-cent stamps. That is, prove that for every integer $n \geq 13$, there exist some non-negative integers $A$ and $B$ such that

$$n = 3A + 7B.$$
3. Consider the following recurrence relation, where $n$ is a power of 2.

$$T(n) \leq \begin{cases} 
0, & n = 1 \\
2T(n/2) + \log n, & n > 1.
\end{cases}$$

Prove by induction that

$$T(n) \leq An + B\log n + C$$

and determine the constants $A, B, C$. 

4. Consider a $2^n \times 2^n$ board, with one of its four quadrants missing. That is, the board consists of only three quadrants, each of size $2^{n-1} \times 2^{n-1}$. Let’s call such a board a quad-deficient board. For $n = 1$, such a board becomes an L-shape 3-cell piece called a tromino, as shown below.

(a) Use a divide-and-conquer technique to prove by induction that a quad-deficient board of size $2^n \times 2^n$, $n \geq 1$ can always be covered using some number of trominoes. (By covering we mean that every cell of the board must be covered by a tromino piece, and the pieces must not overlap.) Use diagram to help describing your algorithm and proof.

(b) Let $f(n)$ be the number of tromino pieces used for covering a $2^n \times 2^n$ quad-deficient board. Write a recurrence for $f(n)$.

(c) Illustrate the covering produced by the algorithm for $n = 3$ (that is, $2^3 \times 2^3$ board).
5. (a) Insert the following sequence of elements into a Binary-Search-Tree (BST), starting with an empty tree: (50, 90, 200, 25, 20, 10, 65, 35, 250).

(b) Delete element 80 in the following BST. (First complete the picture by carefully drawing a line from each node to its children, to get a valid BST.)

(c) What is the worst-case and average-case time complexity of BST operations SEARCH, INSERT, and DELETE?