Computational Assignment 1, Section 222, Fall 2001

Euler's Method

Due in class on Wednesday, October 3, 2001

a) Solve the Initial Value Problem (IVP) to obtain the exact solution $y(x)$:

$$\frac{dy}{dx} = \frac{1}{2}(y-1)^2, \quad y(0) = 2$$

b) In this part you will apply the Euler method to numerically approximate the solution you found in part **a)**. You should write a program (in whatever programming language you are familiar with) that implements this method. Your program should save x_n, y_n, $y(x_n)$ and $|y_n - y(x_n)|$ into a file so that you can subsequently make the graphs required below.

Use $h = 1/N$ over the $x-$interval $[0, 1]$, where N is an integer you will set. First, test your program by starting with $N = 25$ in order to verify, by successively doubling N, that the approximate solution approaches the exact solution you found in part **a)**. Produce graphs that look like Figures 6.1.6 and 6.2.9 in the textbook to make sure your program works correctly. I, for example, would solve this problem with $N = 25$, $N = 50$, $N = 100$, and $N = 200$, and then graph all those results together with the exact solution found in part **a)**. If I don't see the numerical solution becoming identical to the exact solution as I make h smaller and smaller (by increasing N) then I know I have a mistake in my program.

c) Graph the cumulative error, $|y_n - y(x_n)|$ versus x_n, obtained with Euler's method over the interval $[0, 1]$ for $N = 50, 100, 200$. You should produce one graph which should have three curves of the cumulative error, one curve for each value of N. Label each curve to indicate the corresponding value of N. Report the order of accuracy of the method by examining this graph.

d) For Euler's method, determine the N required so that the cumulative error will not exceed 10^{-8} over the interval $[0, 1]$.