Math 712, **Final Project**, Fall 2005
**Due Wednesday, December 21, by 10 AM**

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**1)** Use your Peaceman-Rachford and Mitchell-Fairweather ADI codes with homogeneous Dirichlet boundary conditions on a large domain, $(x, y) \in [-100, 100] \times [-100, 100]$, to solve the two-dimensional Fishers equation

$$u_t = D(u_{xx} + u_{yy}) + ru(1 - u)$$

with initial condition given as follows:

$$u(x, y, 0) = 1; \quad 0 \leq r = \sqrt{x^2 + y^2} < 1$$

$$u(x, y, 0) = e^{-a(r-1)}; \quad r = \sqrt{x^2 + y^2} > 1$$

where $a$ is an input constant that determines the initial condition's exponential decay rate at large $r$ (and hence selects the traveling-wave speed per the discussion in the 3-page handout). Treat the nonlinear term explicitly. If you want, search on the web or in textbooks/handouts for a better way to discretize the nonlinear terms.

1. First test your codes with $D \neq 0$, $r = 0$ to verify your numerical solutions decay to zero for large time.

2. Pick $a$ so that a traveling wave with a certain wave speed is obtained in the continuum. This speed will depend on what you choose for $D$ and $r$, as well as on $a$ per the discussion in the handout. By plotting the $u = 0.5$ level curve of the numerical solution from both schemes at various times make sure that your code correctly simulates the expected traveling wave and its corresponding speed; you will have to use a small mesh size $h$.

3. Once you're sure everything works, produce plots of the $u = 0.5$ level curve of the numerical solution at various times for $h$ varying between large values (say $h = 1$) and small values. Note the differences between the two schemes. Explain these differences by doing a two-dimensional dispersion analysis of the schemes and noting the dependence of the numerical frequency on the polar angle $\phi$ (which is the direction of a particular plane wave mode).

**2)** Make $a$, in the initial condition above, be a function of the polar angle $\phi$ on the plane ($-\pi < \phi \leq \pi$), i.e., now $a = a(\phi)$. Compute with the best scheme, as decided above, with small mesh sizes and see what you get. It will be best to visualize the results by doing color contour plots of the solution. Do not print every single plot you do in the color printer, do all the work first and then decide which plots to actually print for inclusion with your project. NOTE: the handout delineates the range for $a$ that results in wave speeds different from 2 (in the problem scaled so that $D = r = 1$); pick your function $a(\phi)$ so that in some directions the wave speed is the minimum possible. Nobody really knows what to expect; this part could form a nice project to search for patterns in a scalar reaction-diffusion equation.