Math 451, **HW 1**, Spring 2005
DUE IN CLASS: Tuesday, February 15
Chapters 1 and 2 of the textbook are useful, as well as the Math 473 textbook by Strogatz

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Problem #1

Consider the Lotka-Volterra model

$$\begin{aligned} u' &= u(1-v) \\ v' &= \alpha v(u-1) \end{aligned} \tag{1}$$

where $\alpha > 0$ is a real parameter. Do the following:

- Determine the critical points and the equilibrium solutions of (1).
- Do a stability analysis of the equilibrium solutions and characterize the critical points using standard nonlinear dynamics language.
- Now take $\alpha < 0$ and repeat the previous task.

Problem #2

Consider the following (autocatalytic) chemical reaction

$$\begin{aligned} &A + X \to 2X \quad ; \quad k_1 \\ &X + Y \to 2Y \quad ; \quad k_2 \\ &Y \to Z \quad ; \quad k_3 \end{aligned} \tag{2}$$

in a flow reactor (see the textbook for explanation) that keeps $[A] = A_0 > 0$ for all time and removes Z continuously. Let the initial concentrations of the other reactants be $[X](0) = X_0,\ [Y](0) = Y_0$. All rate constants are positive, and all concentrations are non-negative (≥ 0, by definition of concentration). Do the following:

- What is autocatalysis ? Give an example that is simpler than (2) to illustrate your description.
- Write the 2×2 system of ordinary differential equations that governs $[X](t)$ and $[Y](t)$. Determine the critical points and the stability of the equilibrium concentrations in terms of the rate constants and A_0. Use the standard language to characterize the critical points.
- Is it possible for $[X]$ and $[Y]$ to be completely consumed in this reaction ? What sort of behavior in time is exhibited by $[X]$ and $[Y]$? Your analysis in the previous part provides answers to these questions.
- Use PPLANE7 to draw the phase plane for this system assuming some values for the rate constants and A_0.

Problem #3

For the following system,

$$\begin{aligned} x^{'} &= y + \mu x \\ y^{'} &= -x + \mu y - x^2 y, \end{aligned} \tag{3}$$

a Hopf bifurcation occurs at the origin when $\mu = 0$. Do the following:

- What is a "limit cycle" ? What is a "Hopf bifurcation" ? What is a "subcritical Hopf bifurcation" ? What is a "supercritical Hopf bifurcation" ? Illustrate your answers.
- Analyze the critical point at the origin.
- Using PPLANE7, plot the phase portrait of (3) and determine whether the bifurcation is subcritical or supercritical (you will have to plot phase planes with $\mu < 0$ and with $\mu > 0$).