Math 712, **Homework Set 2**, September 12, 2005
Due Monday, September 19

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1. Compute the symbol of the fourth-order accurate centered-difference approximation to u_x, and use it to derive an estimate for the number of points/wavelength in terms of the relative error, $e_4(kh)$, in Fourier space.

2. In this problem we consider $u(x) = \sin kx$ on a periodic domain of length $L = 2\pi$. Use the code *hw2a.f* to verify the accuracy of the second- and fourth-order accurate approximations to u_x by producing one graph of the error for $k = 1$ and $k = 5$ (the graph should have four straight lines on it), and computing the slope. Then, comment out the *dimension* line, uncomment the line immediately below it, and redo the graph. Explain **a)** why the two graphs are different, **b)** why at least for $nmax = 1500$ (smallest $h = 2\pi/1500$) the fourth-order method breaks down first(change nmax to a larger value and you should see the second-order method breaks down too). It seems that the fourth-order method breaks down at a fixed value of kh (the same should happen with the second-order method at a larger nmax): Can you think of a reason ?

3. In this problem we consider $u(x) = e^{-\sigma^2(x-\pi)^2}$ on a periodic domain of length $L = 2\pi$. This function has a known Fourier transform $\hat{u}(k) = \frac{1}{\sigma\sqrt{2}}e^{-k^2/4\sigma^2}$; many Fourier modes contribute to make up the Gaussian function on the real line.

a) Use the code *hw2b.f* with $\sigma = 6$ and $\sigma = 20$, and the DFT capability of the plotting program *xmgr* (found under its menu: $Data \rightarrow Transformations \rightarrow Fourier\ transforms$) to determine an h such that the transform of u_m is close to $\hat{u}(k)$ for $|k| \leq \pi/h$. Explain why $h_{\sigma=6} > h_{\sigma=20}$. In this part, you are essentially finding (by doing pictures) the appropriate grid size to eliminate aliasing in the Fourier transform of the evaluated function values.

b) Using the mesh sizes from part **a)**, produce graphs of the spectrum to explain the large error reported by the code in using $D_0(h)u_m$ and $D_4(h)u_m$ to approximate u_x (e.g., place the DFT's of $D_0(h)u_m$, $D_4(h)u_m$, and $u_x|_m$ on the same graph). Use the code to determine the $h_{\sigma=10}$ and $h_{\sigma=20}$ so that the spectrum of the approximate derivatives are close to the spectrum of the exact derivative (now, the derivative error in the discrete L_2 norm should be small).