Math 707, **Homework Set 4**, SPRING 2002
DUE: March 28, 2002.

Write two programs, one using the standard Runge-Kutta $O(\Delta t^4)$ accurate time-integration scheme and the other using the reduced-storage version of the Runge-Kutta time-integration scheme (given in class with $d = 4$), to solve the $2\pi-$periodic Burger's initial value problem stated below. Space derivatives are to be computed on a spatial grid with N Fourier collocation points using the subroutine FDERV distributed for HW3. You should use double-precision arithmetic.
The Burger's periodic initial value problem in question is:

$$u_t + uu_x = \nu u_{xx} \quad ; \quad x \in (0, 2\pi), \ t > 0, \tag{1}$$

subject to the boundary and initial conditions:

$$u(0,t) = u(2\pi, t) \quad ; \quad t \geq 0, \tag{2}$$

$$u(x,0) = f(x) \quad ; \quad x \in (0, 2\pi). \tag{3}$$

A periodic solution of (1)-(2) with speed c is (via the Cole-Hopf transform):

$$u(x,t) = -2\nu \frac{\frac{\partial \phi}{\partial x}(x - ct, t+1)}{\phi(x - ct, t+1)}, \tag{4}$$

where

$$\phi(x,t) = \sum_{n=-\infty}^{\infty} e^{-\frac{(x-(2n+1)\pi)^2}{4\nu t}}. \tag{5}$$

The parameters c and ν must be inputs to the programs. For each program use (4)-(5) to set the initial condition $f(x)$ in (3).

Take $\nu = 0.2, \ c = 4$. Run your programs to compute the solution of (1)-(3) at $t = \pi/8$ for $N = 16, \ 32, \ 64$. At each N determine the timestep so that your solution is as accurate as possible (and stable!) with each of the two time-integration schemes. Produce three graphs, one for each N, showing the exact solution (computed with your programs from (4)-(5)) and the numerical solutions for each time-integration scheme (computed with your programs) at $t = \pi/8$. Then, set $\nu = 0.1$ and repeat what you did for $\nu = 0.2$ to obtain three more graphs. Discuss your results, and determine whether the case $\nu = 0.1$ requires higher resolution (why ? why not ?).