Math 707, **Homework Set 5**, SPRING 2002
DUE: April 11, 2002.

Given a wave $u(x) = \sin Mx$, the error for Fourier pseudospectral differentiation is high when $N < 2M$ (N =number of collocation points in $[0, 2\pi]$) and pretty close to machine accuracy for $N > 2M$ (see lower right-hand figure from OUTPUT 7 in the textbook, or verify this yourself using the code from HW3). This is why one says "for Fourier pseudospectral differentiation you need at least two grid points per wavelength to accurately compute a derivative." Do the following:

- Determine which functions will produce an error in the first derivative that behaves in a similar fashion when Chebyshev pseudospectral differentiation is used, i.e., the error is large up to some N and then drops to machine accuracy when N exceeds a certain value. Demonstrate by producing a graph of $||u' - u'_N||_\infty$ versus N.
- Determine for Chebyshev differentiation the minimum number of grid points per wavelength that is required in order to obtain the derivative of a wave (fix it so it is zero at $x = \pm 1$) with 6-digit accuracy. Explain why this minimum number is not 2.