Computational Project, 2001 Summer Session

Investigation of a Multistep Scheme for the Oscillator Equation

Write a program (in any language you like) to implement the scheme

$$\frac{x_{n+1} - 2x_n + x_{n-1}}{\Delta t^2} + 2\nu\omega\frac{x_{n+1} - x_{n-1}}{2\Delta t} + \omega^2 x_n = 0 \tag{1}$$

to solve the initial value problem $x'' + 2\nu\omega x' + \omega^2 x = 0$, $x(0) = x_0$, $x'(0) = x'_0$, for $t \in [0, 50]$ with fixed $\nu = 0.1$. Consider both $\omega = 1$ and $\omega = 5$ with $x_0 = 1$, $x'_0 = 0$ and $x_0 = 1$, $x'_0 = 1$. Use the exact solution to provide the starting value x_1. Your program should output two files, one for the exact solution and the other for the numerical solution. Each file should contain two columns, one for $t_n = n\Delta t$ and the other for the corresponding $x(n\Delta t)$ or x_n.

- For each set of parameters and initial conditions above use your program to verify both the stability condition we derived in class and the picture we drew on the board (reproduced as Figure 1 below). For this verification you should produce some graphs that can convince an audience of your fellow students (and me!).

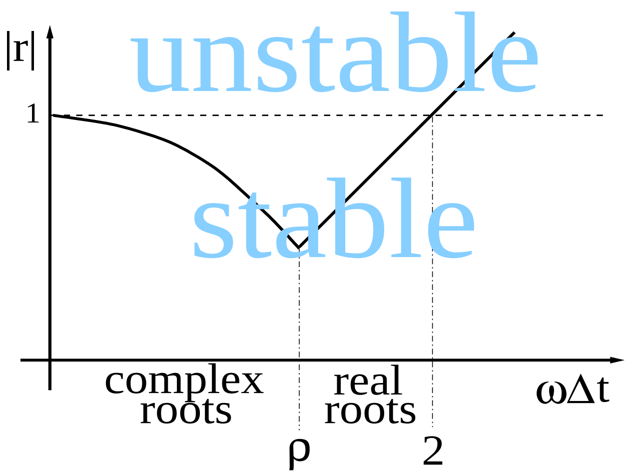

Figure 1: $\rho = 2\sqrt{1 - \nu^2}$.

- Produce a graph of $\frac{\Phi - \bar{\omega}\Delta t}{\bar{\omega}\Delta t}$ for $\omega\Delta t \in [0, 5]$ and $\nu = 0.1$. Verify this graph with your program by graphing on the same axis the actual phase error measured from the numerical output.

- Replace in (1) the approximation $\frac{x_{n+1} - x_{n-1}}{2\Delta t}$ with $\frac{x_{n+1} - x_n}{\Delta t}$, and with $\frac{x_n - x_{n-1}}{\Delta t}$, and derive the stability condition in each case. Which of the three schemes would be preferred as far as the size of the maximum allowed timestep is concerned ?