

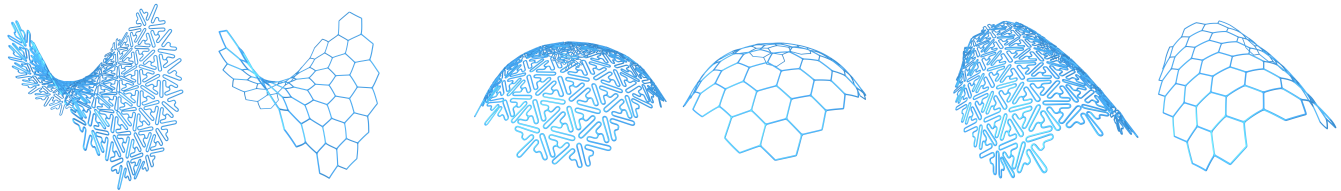
# Simulation of Flexible Patterns by Structural Simplification

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**Figure 1: Three simulated flexible patterns, constrained at four anchor points each. Left: the deformed full pattern. Right: the simplified pattern with appropriate stiffness parameters to mimic the deformation behavior of the full pattern. The three examples demonstrate the possibilities of such meso-structures to approximate different Gaussian curvatures of the surface: hyperbolic, parabolic, and cylindrical.**

## ABSTRACT

In the field of computational design of meta materials, complex patterns of meso-structures have seen increased interest because of the ability to locally control their flexibility through adjustment of the meso-structure parameters. Such structures come with a number of advantages like, despite the simplicity of their fabrication, their ability to nestle to sophisticated free-form surfaces. However, the simulation of such complex structures still comes with a high computational cost. We propose an approach to reduce this computational cost by abstracting the meso-structures and encoding the properties of their elastic deformation behavior into a different set of material parameters. We can thus obtain an approximation of the deformed pattern by simulating a simplified version of the pattern using the computed material parameters.

## CCS CONCEPTS

• **Computing methodologies** → *Shape modeling*.

## KEYWORDS

computational design, fabrication, elastic deformation, meso-structures, meta-materials

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## 1 INTRODUCTION

The popularity of modern additive manufacturing techniques has led to many advancements in the field of computational design of

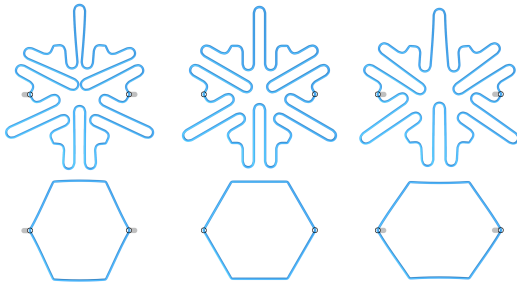
meta-materials. Increased attention has been given to the possibility of building 3D objects out of volumetric structures composed of regular [Panetta et al. 2015] as well as stochastically sampled [Martínez et al. 2017] grids. Another branch are flat flexible material sheets [Malomo et al. 2018] which can be used to form 3D surfaces. Their advantage is additionally that they can be easily adapted to various shapes and easily produced, transported and assembled on site.

In general, the deformation behavior of such materials can be controlled by incorporating meso-structures with the desired mechanical properties [Martínez et al. 2019] that may also form elaborate aesthetically pleasing patterns, which can be used to approximate 3D surfaces well.

## 2 GOALS

However, using such complex patterns for inverse shape design leads to two problems: first, their distribution on the surface is a non-trivial geometric task. Second, there is the problem of performance. When certain parameters of the pattern need to be optimized to achieve a predefined goal, simulation of the physical deformation behavior of the pattern is necessary. If the structure of the pattern is very complex, then the high resolution leads to a significant computational cost. To speed up the optimization process, simplifications need to be made. In this work we address this problem by simplifying the structure of the pattern by encoding its mechanical properties into the material parameters used in the physical simulation by and further performing a homogenization of the meta-material on the coarse level.

Our aim is to further investigate the relationship between the meso-structure parameters of the complex patterns and the material stiffness parameters of the simpler geometric pattern, and furthermore examine the possibility of finding stiffness parameter values that best approximate the behavior of the complex pattern. This further leads to the goal of having simplified homogenized structures which approximate the Gaussian curvature of free-form surfaces well as shown in Figure 1. One disadvantage of spiral-structures proposed by Malomo et al. [2018] is that the density of the patterns on the surface is not homogeneous. Depending on the



**Figure 2: Deformation behavior when the anchor constraints (circles) are moved toward or away from the center. The corner vertices of the simple hexagons (bottom) match their corresponding vertices of the complex hexagons (top).**

underlying distortion of the surface induced by the local surface metric, the coverage may differ significantly. We aim at a coverage which remains approximately uniform across the surface.

In order to do so we develop an optimization scheme which aims at the search of optimal parameters for our meso-structures while the geometric patterns on the surface try to nestle to the target surface curvatures. This formulation is inspired by the work of Jiang et al. [2015] who choose the patterns in accordance with the surface but preserve their planarity. Our goal is balance between better preservation of the original pattern shape with respect to local affine deformations and the behaviour of the homogenized structure.

### 3 OUR APPROACH

In general, as a first step in the design of meta-materials for elastic coverage of free-form surfaces, we must choose which type of pattern and meso-structure to use in our investigations. For the pattern, we decide on a regular tiling of hexagons which allow for regular tessellation of the Euclidean plane. For the meso-structure, we use a type of no-sag springs with so-called *oscillating edges*. To create this meso-structure, we transform each hexagon edge by introducing a natural rest curvature in the underlying 2D plane so that instead of a straight line we obtain a winding curve (see Figure 2 for an example). The number of windings and amplitude of the oscillations are the free parameters of this meso-structure.

There are multiple reasons for using a hexagonal tiling as our pattern. First, hexagons are one of only three polygon types that allow for regular tilings, the others being triangles and rectangles [Jiang et al. 2015]. Second, the even number of edges allows for a consistent orientation of the oscillating edges—when traversing the edges of a hexagon, the middle peak of the oscillating edges always alternates between pointing inward and outward on consecutive edges. Finally, from the perspective of meta-materials, a hexagonal pattern leads to isotropic material behavior [Schumacher et al. 2018]. While rectangles also have the second property, their meta-material behavior only shows a tetragonal symmetry.

Our physical simulation is based on the Discrete Elastic Rods (DER) formulation [Bergou et al. 2010], treating each hexagon edge as a rod. This formulation allows the computation of the elastic stretching, bending and twisting energies of each rod under a set of

constraints, based on the geometry of the pattern, and to find the equilibrium state of the structure by minimizing the elastic energy.

The DER formulation requires stiffness parameters to dictate how much the material of the pattern resists each type of elastic deformation. Usually, these values are set based on empirical experiments with the corresponding real world materials. However, they also enable us to look at the pattern at a lower level of detail, simplifying each meso-structure to a simple straight edge made of a meta-material with different stiffness values instead.

For example, pulling at the ends of an oscillating edge causes in-plane bending in certain regions of the meso-structure. However, when looking at the straight line connecting the ends of the edge, this deformation corresponds to stretching of the line instead. Thus, even if the real-world material of the pattern is inextensible, we would allow the meta-material of the simplified pattern to stretch by choosing appropriate stiffness values.

### 4 RESULTS

The top of Figure 2 shows an example of how a single hexagon cell with meso-structures deforms when moving the two anchor points marked by circles along the x-axis. For the simplified hexagon cell (bottom), this in-plane bending deformation corresponds to both stretching and in-plane bending of the edges. By finding appropriate material stiffness parameters, the coordinates of the corner vertices become identical for both the complex and simplified cells.

Figure 1 shows larger patterns made of  $7 \times 6$  hexagon cells simulated using our approach. The complex patterns on the left contain 9321 vertices and 9263 edge segments, while the simplified patterns on the right only contain 865 vertices and 906 edge segments, significantly reducing the computational cost of the simulation while yielding a good approximation of the resulting deformed patterns.

### Acknowledgements

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