ANALYSIS OF VARIANCE (ANOVA): TESTING DIFFERENCES INVOLVING THREE OR MORE MEANS

REVIEW
Testing hypothesis using the difference between two means:
- One-sample t-test
- Independent-samples t-test
- Dependent/Paired-samples t-test (also known as repeated measures t-test)

WHAT IF WE WANT TO COMPARE MORE THAN TWO GROUP MEANS?
- Can we run multiple t-tests?
- NO! It would lead to alpha inflation:
  - It increases the probability of making a Type I error.
- We need to use a different kind of test:
  - Analysis of Variance (ANOVA)

WHEN CAN WE USE ANOVA?
- We use ANOVA when our Independent Variable (nominal or ordinal) has three or more categories:
  - Race: 1=Black, 2=White, 3=Hispanic, 4=Asian, 5=Other.
  - Opinion on legalization of drugs: 1=Strongly disagree, 2=Disagree, 3=Neutral, 4=Agree, 5=Strongly Agree.
  - Severity of crime: 1=Violation, 2=Misdemeanor, 3=Felony
- And our Dependent Variable is at the interval/ratio level:
  - Income; age; number of arrests; sentence length; rate of crime; etc.

ANOVA ASSUMPTIONS
- A *categorical* (nominal or ordinal) independent variable with three or more groups/categories.
- An *interval/ratio* dependent variable.
- A comparison between three or more independent means.
- Random sampling.
- Normal distribution of DV.
  - This assumption can be relaxed if we have an N>30 for each group.
- Equality of variances.
• WHAT DOES ANOVA TEST?
• Compares the variation or range of scores (measured by the variance) BETWEEN the groups with the variance WITHIN each group.
• If the between variance is much larger than the within variance, the means of different groups will not be equal.
• If the between and within variance are approximately the same size, then there will be no significant difference between the group means.

• THE F-RATIO
• Analogous to t-ratio in t-test.
• The F-ratio is used to determine whether there is a difference in means between at least two groups.
• If the obtained F-ratio is larger than the critical value of F, given x numerator (between) and x denominator (within) degrees of freedom, then there is a statistically significant difference in the means of at least two groups, and we reject the null hypothesis.

• STEPS IN GETTING F-RATIO
• Find mean for each group.
• Find sum of scores, sum of squared scores, sample size, and mean for all groups combined. (Total).
• Find total sum of squares.
• Find the within-group sum of squares.
• Find the between-group sum of squares.
• Find the between-groups degrees of freedom.
• Find the within-groups degrees of freedom.
• Find the within-groups mean square.
• Find the between-groups means square.
• Obtain F-ratio.
• Compare to critical value of F in table.

• EXAMPLE: EFFECT OF TYPE OF PRISON ON DEPRESSION
• A Department of Corrections wants to know whether the type of prison affects inmates’ depression levels.
• DOC administers a depression test to 4 inmates in a low security facility, 4 inmates in a medium security facility, and 4 inmates in a maximum security facility.
  • What are the null and alternative hypotheses?

• EXAMPLE: EFFECT OF TYPE OF PRISON ON DEPRESSION (Steps 1 & 2)
• EXAMPLE: EFFECT OF TYPE OF PRISON ON DEPRESSION
3. Find total sum of squares:

- EXAMPLE: EFFECT OF TYPE OF PRISON ON DEPRESSION

4. Find the within-groups sum of squares:

- EXAMPLE: EFFECT OF TYPE OF PRISON ON DEPRESSION

5. Find the between-groups sum of squares:

- EXAMPLE: EFFECT OF TYPE OF PRISON ON DEPRESSION

6. Find the between-groups degrees of freedom

- EXAMPLE: EFFECT OF TYPE OF PRISON ON DEPRESSION

8. Find the within-groups mean square:

- EXAMPLE: EFFECT OF TYPE OF PRISON ON DEPRESSION

10. Find F-ratio

- SS Within
  - \((132-1)*(13.649)^2 + (114-1)(11.774)^2 + (26-1)(6.754)^2\)
  - \(= (131)(186.30) + (113)(138.63) + (25)(45.62)\)
  - \(= 24405.3 + 15665.19 + 1140.5\)

- SS Between
  - \(= 132(13.7-12.02)^2 + 114(11.1-12.02)^2 + 26(7.54-12.02)^2\)
  - \(= 132(2.82) + 114(0.85) + 26(20.07)\)
  - \(= 372.24 + 96.9 + 521.82\)
  - \(= 990.96\)

- Degrees of Freedom
  - \(Df_{total} = df_{within} + df_{between}\)
  - \(= (N-k) + (k-1)\) where \(N=\) cases and \(k=\) categories
  - \(= (272-3) + (3-1)\)
  - \(= 271\)

- Calculate the Obtained Value
  - \(= \frac{SS_{between}}{(k-1)} / \frac{SS_{within}}{(N-k)}\)
  - \(= MS_b/MS_w\)
    - Only when \(MS_b > MS_w\) can null be rejected
    - No difference between groups, ratio = \(1\)
  - \(= (990.96/2) / (41210.99/269)\)
  - \(-3.23\)
12. Report results:
   - Always report the F ratio, the 2 degrees of freedom, and the alpha level:
     - We reject the null hypothesis. There is a statistically significant difference between at least two of the groups in terms of their average depression levels ($F(2,9)=17.95, p<.05$).
     - When we compare the mean depression level for each group it seems that inmates who are in a maximum security facility have on average (mean=9) higher depression levels, than those individuals who are in medium (mean=8), and low (mean=4) security facilities.
     - However, the F-ratio doesn’t tell exactly between which groups the difference is. To determine that we have to run Post-Hoc tests (SPSS does this).

12. Interpret results:
   - It may be that inmates who are in maximum security facilities have more restrictions on contact with visitors and interaction with other inmates.
   - This isolation may cause higher levels of depression among inmates in maximum security facilities, when compared to inmates in medium and low security facilities.

RUNNING ANOVA IN SPSS

Open *RandStudy data* from Blackboard.

We want to determine whether the perceived likelihood of making lots of money through crime (*PER$CRIM*)

- 1=No Chance
- 2=Small Chance
- 3=Average Chance
- 4=Strong Chance
- 5=Certain

has an effect on the respondents actual criminal income.

What are the Null and Alternative Hypothesis?

RUNNING ANOVA IN SPSS

First test for assumptions:

- IV (*PER$CRIM*) is categorical (ordinal).
- DV is continuous (*CRIM$*).
- We are comparing the means for 5 groups.
- Random sample.
- DV is positively skewed, but we can relax assumption since all groups have $N>30$
- Equality of variances: We ask SPSS to test this assumption
RUNNING ANOVA IN SPSS

• ANALYZE ➔ Compare Means ➔ One-way ANOVA ➔ Move DV (crim$) to “Dependent List” box ➔ Move IV (per$crim) to “Factor” box

• Click on “Options” ➔ Under “Statistics” section check: “Descriptive”, “Homogeneity of Variance Test”
  • If the homogeneity/equality of variances assumption is not met, we can’t report the F ratio because it is biased, and we would use other measures – we would request Brown-Forsythe and Welch.
  • These tests correct for the assumption not being met.

ANOVA SPSS OUTPUT

• The first table shows the descriptive statistics (N, mean, standard deviation, etc) for each group of the IV in terms of the DV

ANOVA SPSS OUTPUT

• The second table shows the results of the homogeneity of variances test.

• Remember that we want this test to be not significant (we want to retain the null hypothesis that all variances are equal).

ANOVA SPSS OUTPUT

• The third table shows the F-ratio results.

• In this case we can see there is a statistically significant difference in the criminal income between at least two groups of perceived likelihood of making lots of money through crime.

ANOVA SPSS OUTPUT

• The last table shows the results of the Post-Hoc test (LSD).

• This test shows exactly between which groups there is a difference in means.

REPORTING THE RESULTS

• The homogeneity of variances assumption is met (Levene(3,401)=3.98, p=.352), therefore the obtained F-ratio is reliable and can be reported.

• We reject the null hypothesis. There is a statistically significant difference in the mean criminal income between at least two groups of the perceived likelihood of making lots of money through crime (F(4,401)=2.388, p<.05).

• When looking at the Post-Hoc test results (LSD) we can determine that the difference in means lies between the group “Strong Chance” (mean=2,145.85) and the groups “Average Chance” (mean=674.44) and “Small Chance” (mean=697.49).

INTERPRETING THE RESULTS

• People who think they have a strong chance of making lots of money through crime will invest a lot of time and energy in criminal enterprises, which will lead them to acquiring more money in illegal ways than people who think their chances of making money through crime are just average or small.
These last two groups of people will probably not make much of an effort to make money through crime, therefore their criminal incomes will be smaller.