Effects of membrane morphology on separation efficiency

Summary of progress made at MPI 2014

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Abstract

The Workshop was asked by Pall Corporation to consider ways in which membrane morphology may be characterized, and how such characterization can be used to assess filtration efficiency. The working group considered a variety of different (but related) approaches, which are outlined below but which will be described in more detail in the forthcoming report.

1 Darcy flow model

In an effort to characterize the effect of membrane variations across the thickness, a simple Darcy flow model was considered first. Taking $x$ to be the variable perpendicular to the membrane (in the direction of flow), and assuming no variation in the plane of the membrane, basic Darcy flow gives

$$u(x, t) = -\frac{k(x, t)}{\mu} \frac{\partial p}{\partial x} , \quad \frac{\partial}{\partial x} (k(x, t) \frac{\partial p}{\partial x}) = 0$$

for the superficial (Darcy) velocity $u(x, t)$. Here $\mu$ is the fluid viscosity, $p(x, t)$ is the pressure, and $k(x, t)$ is the membrane permeability. The key modeling question here is how $k$ evolves in time from its initial state due to fouling. To this end, simple modeling frameworks were developed, based on the assumption that a membrane consists of a periodic array of pores (tubes) of variable cross-sectional radius $a(x, t)$. In the simplest model the permeability $k$ is simply related to $a$, and we model the change in $a$ with time by considering a particle deposition process on the walls of the pore. Particles are advected with the flow in the pore and deposit on pore walls due to some force of attraction (most likely electrostatic). The equations that couple these processes were proposed, and numerical schemes were devised to solve them in situations of (i) constant imposed pressure drop; and (ii) constant imposed flux.

An extension of the model was proposed to account for additional fouling due to “sieving” of large particles, which are too large to pass through membrane pores and which deposit on the membrane upper surface, blocking pores. This was achieved by introducing an additional dependent variable $N(t)$, the number of unblocked pores per unit membrane area. It is assumed that $N$ decreases at a rate proportional to the flux through the membrane, a larger flux being associated with a larger number of carried particles. Blocking of a pore is associated with an additional resistance, added in series with that of the open pore. A numerical scheme will be developed for the augmented model.

2 Particle capture by flow around structures within membrane

The model outlined above contains a parameter related to the deposition rate, but the physics that describe it are not specified. In an attempt to characterize the deposition process, a set of simple model problems was considered. Assuming a membrane to be composed of an interconnected set of simple objects such as spheres and cylinders, we considered the flow of small particles, carried by the filtrate, around such obstacles. Equations describing the concentration of a population of particles, undergoing advection, diffusion and electrostatic interaction with the obstacle, were written down and analyzed. Dimensional
analysis revealed that diffusion is dominated by advection and electrostatic forcing over most of the flow domain, except in some small region around the obstacle (small compared with the obstacle diameter but perhaps large compared with the particle size). The model for particle transport was solved for the simplest canonical obstacle geometry (a sphere) in the case where diffusion is neglected entirely. In this case an expression can be obtained, in terms of simple model parameters relating to the membrane internal structure, for the proportion of particles that are captured by the obstacle. If one considers the membrane to be composed of a periodic lattice of spherical inclusions (for example), such considerations enable one to predict the proportion of particles captured by a membrane consisting of a given number of layers.

3 CFD simulations of particle transport within a structured membrane

An existing Navier-Stokes solver was adapted to solve for flow of filtrate around obstacles which might reasonably represent the membrane internal structure (so far just collections of 2D circular and elliptical obstacles have been implemented in the solver, representational of fibre cross-sections within a fibrous membrane). Under certain assumptions about the behavior of small particles carried by the filtrate, the particle trajectories can also be computed and their capture by the underlying membrane structure predicted under given conditions. In particular, examples of particle capture due to electrostatic attraction between particles and the obstacle(s) have been computed. Such a computational model provides a useful tool to complement the more mechanistic and predictive approaches offered by the simplified models outlined in §§1, 2 above. A more sophisticated variant of this model could easily be developed to account for the change in obstacle shape and size due to the deposition (the fouling process).

4 Considerations of fractal structure of membranes

A search of the relevant literature revealed that other authors (e.g. Yu & Lee [1]) have considered porous media with highly complex internal structure, and have written down explicit expressions for membrane characteristics such as permeability, tortuosity, etc. in terms of the fractal dimension of the membrane (which may itself be calculated from detailed SEM membrane images). Such geometrical considerations can provide a useful framework to characterize clean membranes. Clean membrane characteristics are needed as input to the type of fouling models outlined above in §§1, 2.

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References