The normal and shear stresses in horizontal X, and vertical Y directions at a point are as shown:

(i) Draw the Mohr circle for the state of stress and mark all significant points.

(ii) Determine the magnitude and direction of $\tau_{\text{max}}$

(iii) Determine the magnitude and direction of the principal normal stresses ($\sigma_1$ & $\sigma_2$)

(iv) Determine the normal and shear stresses in an axis system which rotated 15° cw with respect to the original X-Y axis system.

(i) The given stresses are:
In X direction:
$\sigma_x = 500$ MPa; $\tau_{xy} = -100$ MPa (negative because $\tau_{xy}$ in X creating a CW rotation).
In Y direction:
$\sigma_y = -300$ MPa; $\tau_{xy} = 100$ MPa

Using the $\sigma-\tau$ axis system of Mohr circle, X and Y points show the stress conditions in X and Y direction. CX and CY radii are representing the actual X and Y directions, respectively.

$OC = (\sigma_x + \sigma_y)/2 = (500 - 300)/2 = 100$ MPa

Radius = $CX = CY = [(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2]^{0.5} = \left[\left(\frac{500+300}{2}\right)^2 + 100^2\right]^{0.5} = 412$ MPa

$\angle DCX = \tan^{-1} \left[ \frac{DX}{DC} \right] = \tan^{-1} \left[ \frac{DX}{(OD-OC)} \right] = \tan^{-1} \left[ \frac{100}{(500-100)} \right] = 14^\circ$

(ii) The orientation of $\tau_{\text{max}}$ in Mohr circle with respect to CX = 14+90 = 104° ccw
So in the actual element $\tau_{\text{max}}$ will occur at an orientation 104/2 = 52° cw from the original X direction. The magnitude of $\tau_{\text{max}}$ = the radius = 412 MPa
(iii) Maximum principal normal stresses are $\sigma_1$ and $\sigma_2$.

$\sigma_1 = OC + \text{radius} = 100 + 412 = 512 \text{ MPa (tensile)}$

$\sigma_2 = OC - \text{radius} = 100 - 412 = -312 \text{ MPa (compressive)}$

In Mohr circle $\sigma_1$ is $14^\circ$ ccw from CX. So in reality $\sigma_1$ will occur at an angle $14/2 = 7^\circ$ cw from the original X direction.

The direction of $\sigma_2$ will be perpendicular to $\sigma_1$.

(iv) Given that the new axis system (U–V) in reality is rotated $15^\circ$ cw from X–Y axis system. In Mohr circle the diameter UV is drawn $15*2 = 30^\circ$ ccw from CX. The radius CU represents the new U direction and the radius CV represents the new V direction. The coordinates of the points U and V represent the normal and shear stresses in U and V directions respectively.

$\angle UCD = 30 - 14 = 16^\circ$

At U, normal stress $\sigma_u = OC + CU*\cos(16^\circ) = 100 + 412*\cos(16^\circ) = 496 \text{ MPa}$

At V, normal stress $\sigma_v = OC - CU*\cos(16^\circ) = 100 - 412*\cos(16^\circ) = -396 \text{ MPa}$

$\tau_{uv} = CU*\sin(16^\circ) = 412*\sin(16^\circ) = 113 \text{ MPa}$, with shear stress in U direction being positive.