A plate is made up of AISI 304 stainless steel which has the following material property: Yield strength = 200 MPa, ultimate tensile strength = 552 MPa, modulus of elasticity = 193 GPa. If the plate is subjected to the following combinations of static stresses, determine the factor of safety achieved in each of the following cases.

(i) $\sigma_x = 150$ MPa, $\sigma_y = 0$ MPa, and $\tau_{xy} = 0$ MPa, according to the maximum shear stress theory.

(ii) $\sigma_x = 150$ MPa, $\sigma_y = 0$ MPa, and $\tau_{xy} = 0$ MPa, according to the maximum distortion energy theory.

(iii) $\sigma_x = 150$ MPa, $\sigma_y = -50$ MPa, and $\tau_{xy} = 0$ MPa, according to the maximum shear stress theory.

(iv) $\sigma_x = 150$ MPa, $\sigma_y = -50$ MPa, and $\tau_{xy} = 0$ MPa, according to the maximum distortion energy theory.

Formulæ used:

$$\tau_{\text{max}} = \text{radius} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \text{center distance of Mohr circle} + \text{radius}$$

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \text{center distance of Mohr circle} - \text{radius}$$

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

(i) and (ii) SOLUTION

For the given stress condition, $\sigma_x = 150$, $\sigma_y = 0$ and $\tau_{xy} = 0$:

From the above formulæ:

$$\tau_{\text{max}} = \sigma_x/2 = 150/2 = 75 \text{ MPa}.$$  
$$\sigma_1 = \sigma_x = 150 \text{ MPa},$$  
$$\sigma_2 = \sigma_y = 0.$$  

You can also determine $\tau_{\text{max}}$, $\sigma_1$ and $\sigma_2$ easily by drawing a Mohr circle as shown.
(i) According to the maximum shear stress theory, $\tau_{\text{max}}$ is compared with shear strength $\sigma_{yp}/2$.
Thus the factor of safety (FOS) achieved = $(\sigma_{yp}/2)/\tau_{\text{max}} = 100/75 = 1.33$

(ii) According to the max distortion energy theory, $\sqrt{\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2}$ is compared with the limiting $\sigma_{yp}$ to detect failure.
$$\sqrt{\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2} = \sqrt{150^2} = 150\text{MPa}$$
The limiting strength value = $\sigma_{yp} = 200$ MPa
The FOS = $200/150 = 1.33$

NOTE: For uniaxial stress, the FOS is same from both the failure theories.

(iii) and (iv) SOLUTION

For the given stress condition, $\sigma_x = 150$ MPa, $\sigma_y = -50$ MPa, and $\tau_{xy} = 0$ MPa:
Using the either formulae or using a Mohr circle:
$$\tau_{\text{max}} = (\sigma_x - \sigma_y)/2 = (150+50)/2 = 100\text{ MPa}.$$  
$$\sigma_1 = \sigma_x = 150\text{ MPa}, \text{ and}$$
$$\sigma_2 = \sigma_y = -50.$$  

(iii) According to the maximum shear stress theory
FOS = $(\sigma_{yp}/2)/\tau_{\text{max}} = 100/100 = 1$

(iv) According to the maximum distortion energy theory
$$\sqrt{\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2} = \sqrt{150^2 + 150 \times 50 + 50^2} = 180.3\text{MPa}$$
FOS = $\sigma_{yp}/180.3 = 200/180.3 = 1.11$

NOTE: For a bi-axial stress condition the FOS is different from the two failure theories
2. The critical stresses on a component are $\sigma_x = -80$ MPa, $\sigma_y = 0$ MPa, and $\tau_{xy} = 100$ MPa. It is made up of cast iron with ultimate compressive strength $= 165$ MPa. If a factor of safety $= 2$ is required for this component, would the component be safe to use? Show your calculations.

As cast iron is a brittle material, we use maximum normal stress theory to predict failure.

The magnitude of the maximum normal compressive stress can be found by using the formulae given in (1) or by drawing a Mohr circle.

$\sigma_2 = \text{center distance of Mohr circle} - \text{radius}$

$$\sigma_2 = \left(\frac{\sigma_x}{2}\right) - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \left(\frac{-80}{2}\right) - \sqrt{\left(\frac{-80}{2}\right)^2 + 100^2} = -147.7 MPa$$

With a FOS $= 2$, allowable stress in compression

$\sigma_{\text{allowable}} = \frac{\sigma_{\text{ult}}}{\text{FOS}} = \frac{165}{2} = 82.5$ MPa (compressive)

Because the maximum normal compressive stress $\sigma_2 = 148$ MPa is more than the $\sigma_{\text{allowable}} = 82.5$ MPa, the component can fail at the given stress level.