What is biomechanics?

Biomechanics is the field of study that makes use of the laws of physics and engineering concepts to describe motion of body segments, and the internal and external forces, which act upon them during activity.

One objective of biomechanics is to determine the internal forces in muscles, tendons, bones and joints that arise within the human body due to interaction of external forces, gravitational forces on the body segments, and body posture. These internal forces are then used to explain the relationship between external environments and the internal injuries and tissue stresses.

Sports biomechanics explores the relationship between the body motion, internal forces and external forces to optimize the sport performance.

Biomechanics often applies modeling approach to understand the internal reaction forces in a body part of interest. Models of body segments are developed at varying degree of details depending upon the objective of the study. Modeling requires data about the size, mass and inertial properties of human body segments, and tissue strength limits (of muscles, bones, cartilages, tendons, ligaments etc.). Scientists have extensively studied such data on human internal structure and strength employing experimental study, cadaver dissection, X-rays, CT scans, and MRI.

Depending upon the detailing level of biomechanical modeling, it can employ complex mathematical and engineering principles. However, simple biomechanical models of human structure can help us to understand how the internal stresses are developed, and its interaction with the external environment.

Biomechanics uses the principles of mechanics. We will review some basic principles of mechanics before going into biomechanics.

Newton's laws of mechanics

1. A body will maintain its stationary (or moving) state, until and unless, a net force is applied to it that changes its state.
2. When a net force 'F' is applied to a body of mass 'm', it changes the body's acceleration by 'a'. These three quantities are related to each other by the equation F = m * a.
3. Each force has an equal and opposite reaction force.

The standard international (SI) unit of force is one Newton, which is the amount of force required to change the acceleration by 1 metre/second^2 of a mass of 1 kg.

Force vector

Force is a vector quantity, that is, it requires two quantities to describe it. One is the magnitude of the force and the other is the direction to which it is acting. Examples of other vector quantities are velocity and acceleration. As opposed to this, mass and distance are scalar quantities because they need only magnitude to describe them but no direction is necessary.

Graphically a force is represented by an arrow, the length of the arrow represents the magnitude and the direction of the arrow provides the direction of the force.

Resultant of multiple forces

Since the force have magnitude and direction, the magnitude of the forces acting on a point in the same direction **only** can be added numerically to obtain the magnitude of the resultant force. But if the forces have different directions, they cannot be added numerically to obtain the magnitude of the resultant force. Figure 1 below shows the concept in a two dimensional situation. If the two forces 2N and 4N acts on the square block on one point and are parallel to each other then, the equivalent amount of force, which is called resultant force acting on the block at the same point, is 6N. But if the forces 2N and 4N are acting on different directions, then their resultant will not be 6N.

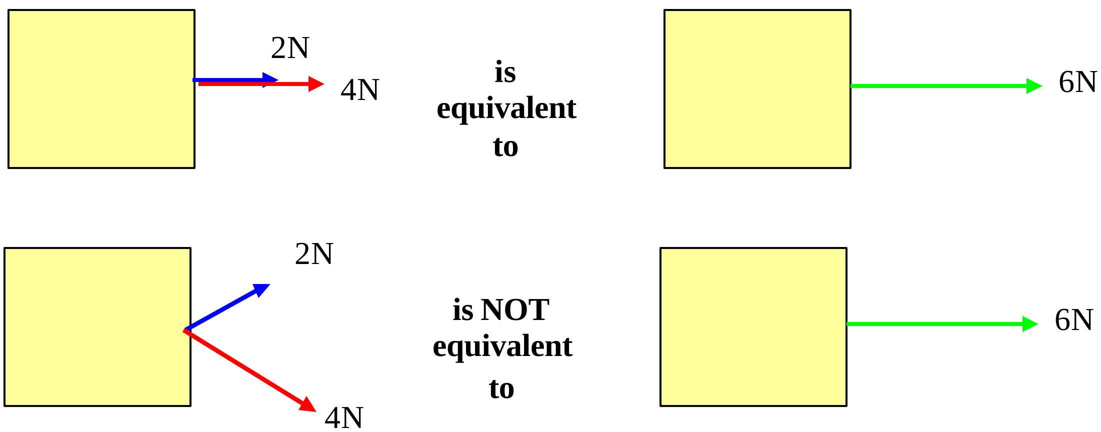


Figure 1. Forces are vector quantity. Their magnitudes cannot be numerically summed up to get the magnitude of the resultant force, unless they are concurrent and parallel.

Force parallelogram

When two forces are not parallel to each other, we can draw a force parallelogram to obtain the magnitude and direction of the resultant force. Two parallel lines to the force vectors F1=4N and F2=2N are drawn graphically to form a parallelogram (Figure 2). The diagonal line of the parallelogram from the point of action of the two forces represents the resultant force vector. If the force vector lines are drawn to the scale then the magnitude and direction of the resultant force vector can now be measured directly from the graphics.

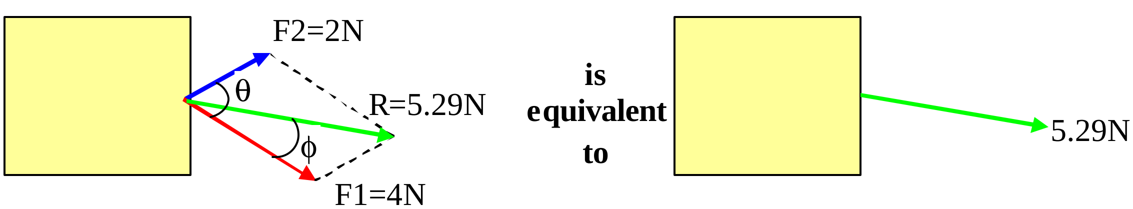


Figure 2. Force parallelogram method of finding resultant of two concurrent forces.

If there are three force vectors acting on a point (concurrent), then by taking any two forces we can first find the resultant of the two. Now using the resultant force vector of the two and the third force vector, we can draw a new parallelogram. The diagonal of this parallelogram would then represent the resultant of the three forces.

The magnitude and direction of two concurrent forces can also be determined from the formula given below. This formula can be derived using basic geometry. In the formula, θ is the angle between the two forces and ϕ being the angle between the resultant force and the F_1 force (See Figure 2). Let us apply the formula to find the resultant for our previous example forces of 4N and 2N. Let us also assume that the angle between the two forces θ = 60°.

$$R = \sqrt{F_1^{\,2} + F_2^{\,2} + 2F_1F_2\cos(q)} = \sqrt{2^2 + 4^2 + 2*2*4*\cos(60)} = \sqrt{4+16+16*1/2} = \sqrt{28} = 5.29N$$

$$f = \tan^{-1}\left(\frac{F_2\sin(q)}{F_1 + F_2\cos(q)}\right) = \tan^{-1}\left(\frac{2\sin(60)}{4+2\cos(60)}\right) = \tan^{-1}\left(\frac{2*\sqrt{3}/2}{4+2*1/2}\right) = \tan^{-1}(0.346) = 19.1^{\circ}$$

The magnitude of resultant R came out to be 5.29 N and the angle ϕ of the resultant from the F1 force (F1 = 4N in this case) came out to be 19.1°.

The above method of graphically drawing force parallelogram can be time consuming and complicated. Also the formula method allows us only to add two forces at a time. What if there are more than two forces acting? ***The following two sections describe a much easier method of adding two or more forces.***

Resolving forces into two orthogonal components

Any force vector F can be resolved into two orthogonal (perpendicular) components. Usually the two orthogonal directions are denoted as X and Y, and X being the horizontal and Y being the vertical direction. These two directions X and Y are also called X axis and Y axis. The simple formulae for the resolution are $F_x = F\cos(q)$ *and* $F_y = F\sin(q)$. **Here,** q **is the angle of the force vector F from positive X axis.** We have to follow a sign convention for specifying this angle. When the angle from the positive X axis to the force is counterclockwise (CCW), the angle is positive. If it is clockwise (CW) the angle is negative.

In our previous example the force F1 = 4N makes an angle 30° CW from the positive X-axis. This means the angle θ1 = -30°. Applying the above formula, the X and Y components of this force are:

$F1_x$= 4cos(-30°) = 4(0.866) = 3.46 N
$F1_y$= 4sin(-30°) = 4(-0.5) = -2 N

$F1_y$ comes out to be negative. That means this force is directed towards the negative Y axis, which is vertically downward.

Similarly, the force F2 = 2N is also resolved into X and Y components. F2 is making an angle 30^o with positive X axis, thus the angle $\theta 2 = 30^o$, and

$$F2_x = 2\cos(30^o) = 2(0.866) = 1.73 \text{ N}$$
$$F2_y = 2\sin(30^o) = 2(0.5) = 1 \text{ N}$$

Figure 3 shows the forces 4N and 2N and their X and Y components. Notice that the X and Y components of a force make two sides of a rectangle, who's diagonal is the force that is being resolved. Rectangle is also a parallelogram. This means that the vector sum of the X and Y components of a force will produce the resultant equal to the force. That means a force is equivalent to the vector sum of its X and Y components, and vise versa.

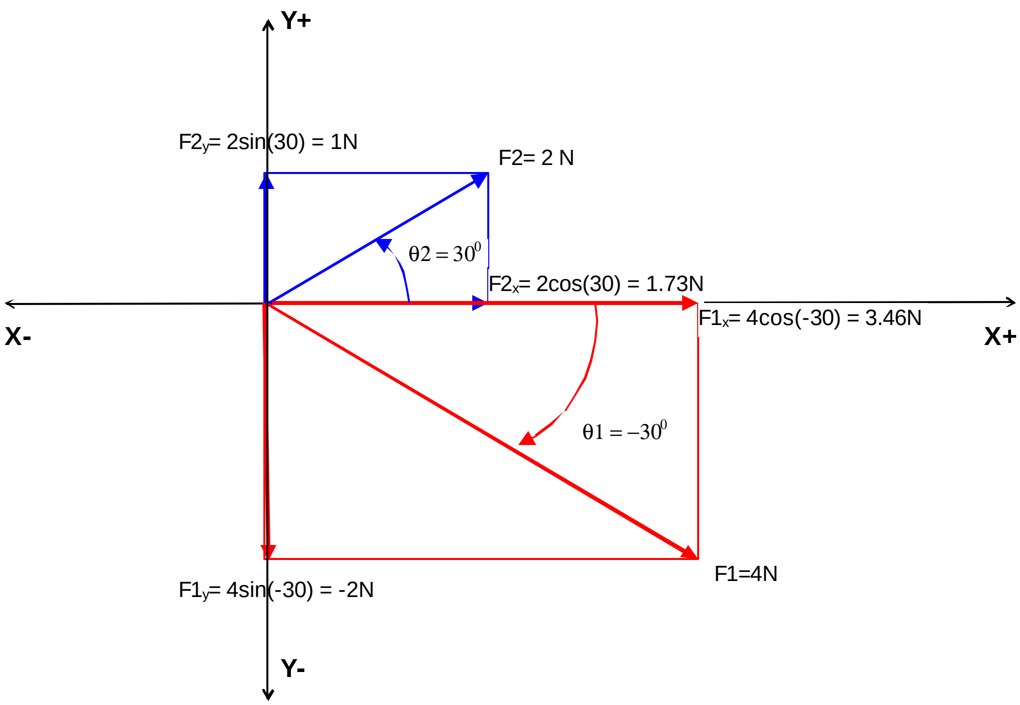


Figure 3. Resolving forces into its X and Y components

Finding resultant of a system of forces from their X and Y components

If there is a system of concurrent forces, we can resolve each one into its X and Y components. Now as all X components are now in the same direction, we can numerically add their magnitudes together to obtain the resultant X component. We can treat all Y components in the same way.

$F_x = F1_x + F2_x + F3_x + F4_x + F5_x + F6_x + \ldots\ldots..$

$F_y = F1_y + F2_y + F3_y + F4_y + F5_y + F6_y + \ldots\ldots..$

Finally we can find the resultant F, of all forces, by finding the resultant of the Fx, and Fy force. Because Fx and Fy forces are at 90^o to each other, the formulae for the magnitude F of the resultant and direction ϕ becomes (angle of the resultant from X axis) much simpler.

$$F = \sqrt{F_X^2 + F_Y^2} \quad and \quad f = \tan^{-1}\frac{F_Y}{F_X}$$

For our example problem,

$Fx = F1_x + F2_x = 4\cos(-30^o) + 2\cos(30^o) = 4(0.866) + 2(0.866) = 3.46 + 1.73 = 5.19$ N

$Fy = F1_y + F2_y = 4\sin(-30^o) + 2\sin(30^o) = 4(-0.5) + 2(0.5) = -2 + 1 = -1$ N

Then,

$$F = \sqrt{F_X^2 + F_Y^2} = \sqrt{5.19^2 + (-1)^2} = \sqrt{26.9361 + 1} = \sqrt{27.9361} = 5.29N$$

$$and \quad f = \tan^{-1}\frac{F_Y}{F_X} = \tan^{-1}\frac{(-1)}{5.19} = \tan^{-1}(-0.1927) = 10.91^\circ$$

Graphically the above operations are shown in Figure 4. The figure also shows that if we find the resultant by the parallelogram method or by resolution method we get the same resultant force F = 5.29 N.

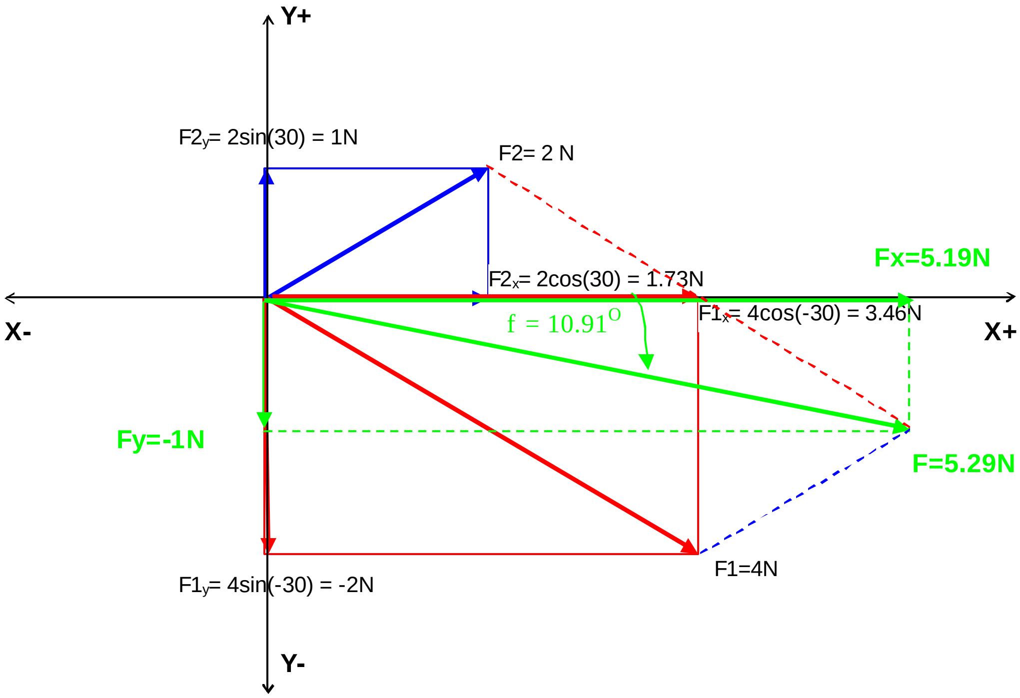


Figure 4. Vector sum of two forces from the resolved forces in X and Y directions.

<u>Moment of a force about a point</u>

If a force F acts on a body at a point A (figure 5a & 5b), then it produces a moment, M = $\bar{d}$ *F around another point B, which is at a (perpendicular) distance $\bar{d}$ from the line of action of the force. If we pin the body at point B, this moment M will try to rotate the body around pin. If the line of action of the force passes through the point B, then $\bar{d}$ =0 and the moment becomes zero. Thus moment of a force about a point on the line of action of the force is always zero.

Moment is also a vector quantity. In two dimensions, it is either CCW or CW. If one is treated as positive the other becomes negative.

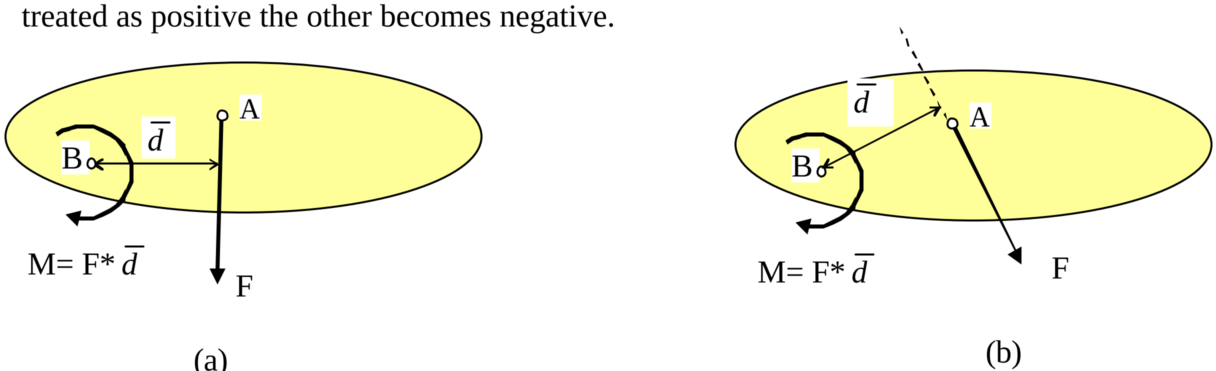


Figure 5. Moment of a force

For calculating moment of an inclined force, it is customary to resolve the force into X and Y components and then to find the moments due to Fx and Fy. If we know the horizontal and vertical distances between the points A and B, say x and y respectively then the moment at point B becomes:

$$M = M_x + M_y = -y*F_x + x*F_y$$

Suppose a force F = 10 N passes through the point A, whose horizontal (x) and vertical (y) distances from point B is 5 and 2 meters, respectively. F is inclined 60^o CW with positive X axis. According to our sign convention of angles it is a negative angle. Then the magnitude of Fx and Fy are

F_x= 10cos(-60^o) = 10(0.5) = 5 N
F_y= 10sin(-60^o) = 10(-0.866) = -8.66 N
$M = M_x + M_y = -y*F_x + x*F_y = (-2*5)+5*(-8.66) = -10 – 43.3 = -53.3$ N-m.

Thus the moment of this inclined force at point B is 33.3 N-m in CW direction.

The above method of calculation of moment is explained graphically in Figure 6.

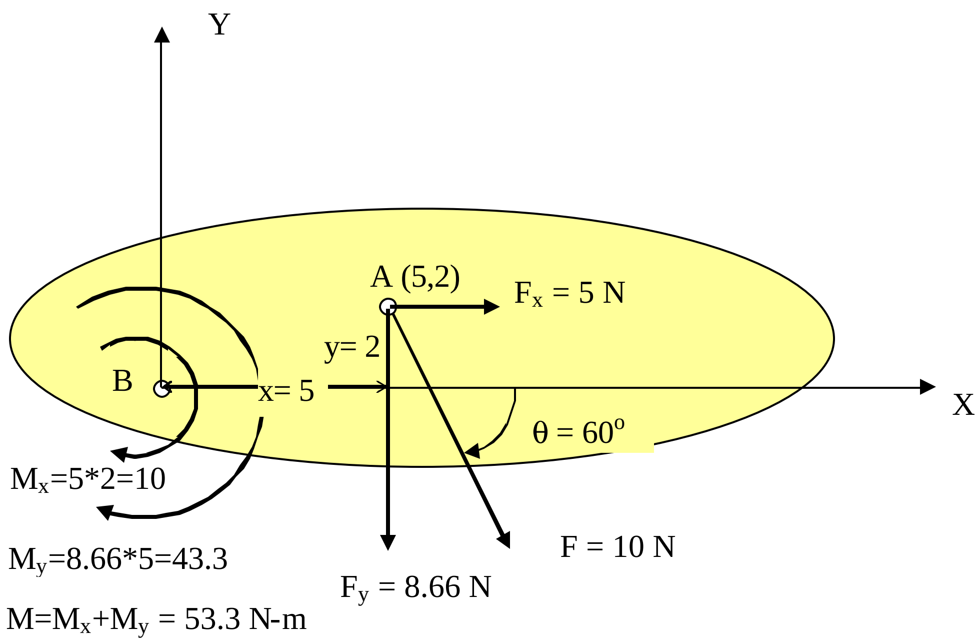


Figure 6. Calculation of the moment of an inclined force from its X and Y components

Condition of static equilibrium and Free body diagram

If a net force acts on a static body, then according to Newton's laws of motion, the body must start moving. If the body doesn't move, it means that some internal forces and moments have developed, that are opposing the external forces and are keeping the body static. **These internal forces and moments that are developed due to the external forces and moments, and that resist any motion of a static body, are called reaction forces and reaction moments, respectively.**

Essentially, to maintain static equilibrium of a body both the following conditions must be satisfied.

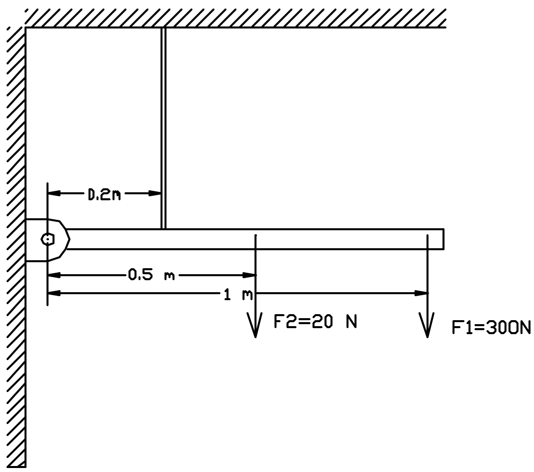
(i) The sum total of all forces (both external and internal) acting on a body in any given direction must be equal to zero. In a 2D situation, this gives us two conditions that must be met.

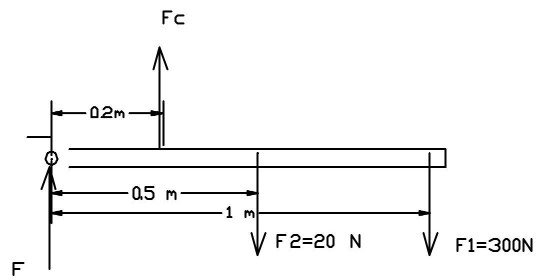
(a) $\sum F_x = 0$ and (b) $\sum F_y = 0$

(ii) The sum total all moments (both due to external and internal forces) about any point on the body must be equal to zero. That is $\sum M = 0$.

For static conditions, most of the times, we can draw a free body diagram of the object we want to analyze. Free body diagram contains all known external forces and unknown internal forces. Then we use the equilibrium conditions, and solve for the unknown internal (reaction) forces and moments.

Example 1. A beam of 20 N weight is attached to a wall by means of a hinge and is supported by a cable from the roof. At the free end, it supports a vertically downward force of 300 N. The pertinent dimensions of the beam are shown in Figure below. If the beam is rigid, what forces will be developed at the hinge and the cable.





Solution: We can guess that the hinge and cable is giving upward forces on the beam, which are the internal forces with unknown values. We draw a free body diagram of the beam, with both known and unknown forces then apply the equilibrium conditions.

(i) $\sum F_x = 0$: **It does not yield anything as there is no force in X direction.**

(ii) $\sum F_y = 0$: **Fc +F-20-300 = 0, i.e. Fc +F = 320** ………(1)

(iii) $\sum M = 0$: **Taking a moment at the left hinge point**

Fc(.2) -20(.5)-300(1) = 0, i.e. .2Fc = 310 i.e., Fc = 310/.2 = 1550N

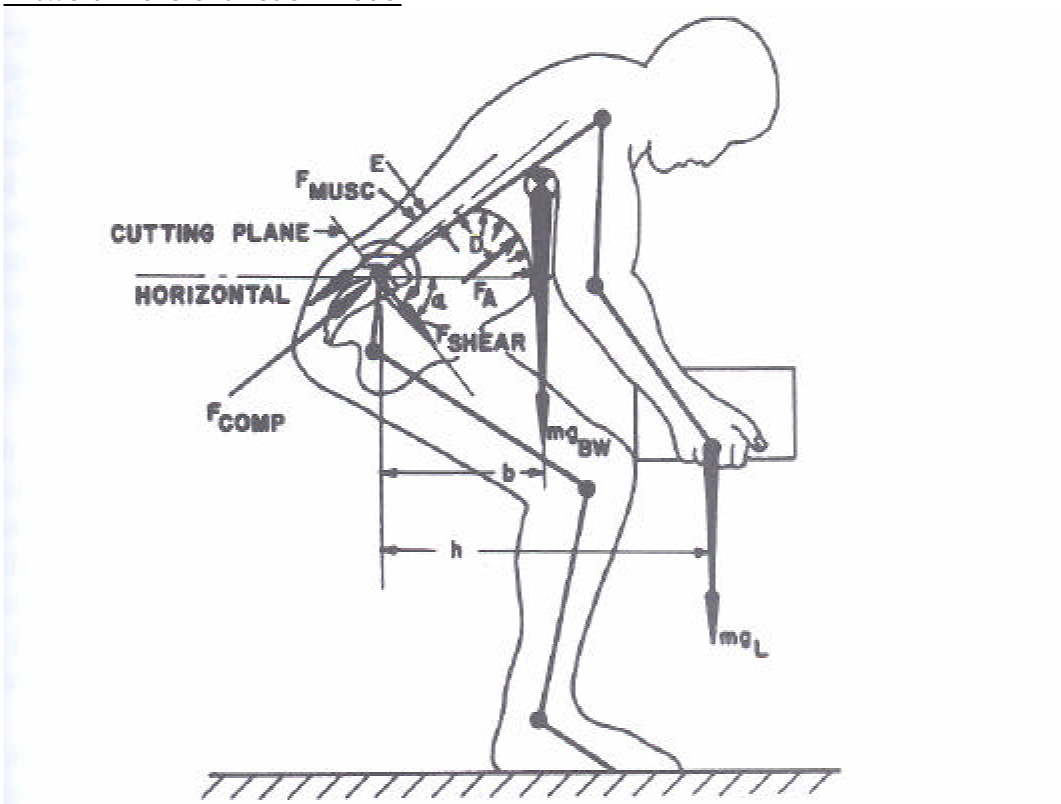
Then from equation (1), **F = 320 -1550 = -1230 N**

We see that the force F at the hinge joint is -1230N. This means the direction of the force is in the negative Y direction that is downward. The cable will apply an upward force of 1150 N.

Body system static models

Using similar approach as described above we can estimate internal reaction forces developed at the body joints, and internal forces developed by the muscles to maintain the static equilibrium.

A two dimensional back model

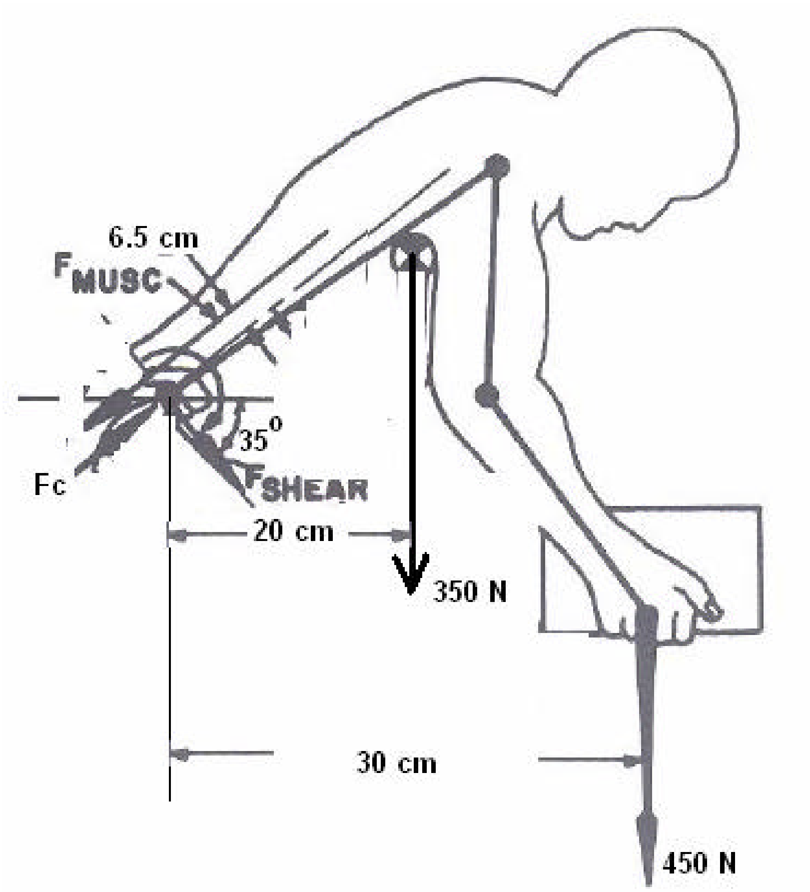


*gure 6.25 Simple cantilever low-back model of lifting, as adapted by Chaffin (1975) for sta-
: coplanar lifting analyses.*

Assume lifting is occurring at the mid-sagittal plane. We want to estimate the back muscle force F_{MUSCLE}, spine compressive force F_{COMP}, and spine shear force F_{SHEAR} at the moment of the lift, when the body angles are similar to as shown above. We also assume only the erector spinae muscle is the sole provider of the counterbalancing moment during this lift. No other back muscles are significantly active. We are also assuming that the F_{SHEAR} is resisted by the passive resistance from the joint capsule and the ligaments at the spine.

Load in hand = 450 N = (450*2.2/9.81 = ~100 lb)
Upper body weight above L5/S1 joint = 350 N
E = Distance of erector spinae muscle from the center of spine = 6.5 cm
h = distance of the load from the center of spine at L5/S1= 30 cm
b = distance of the upper body center of gravity from center of spine at L5/S1 joint =20 cm
α = Upper body angle with horizontal = 55^{o}

Solution: To find out the internal forces, we cut the upper body by a section perpendicular to spine at L5/S1 joint and show all internal and external forces acting on the upper body.



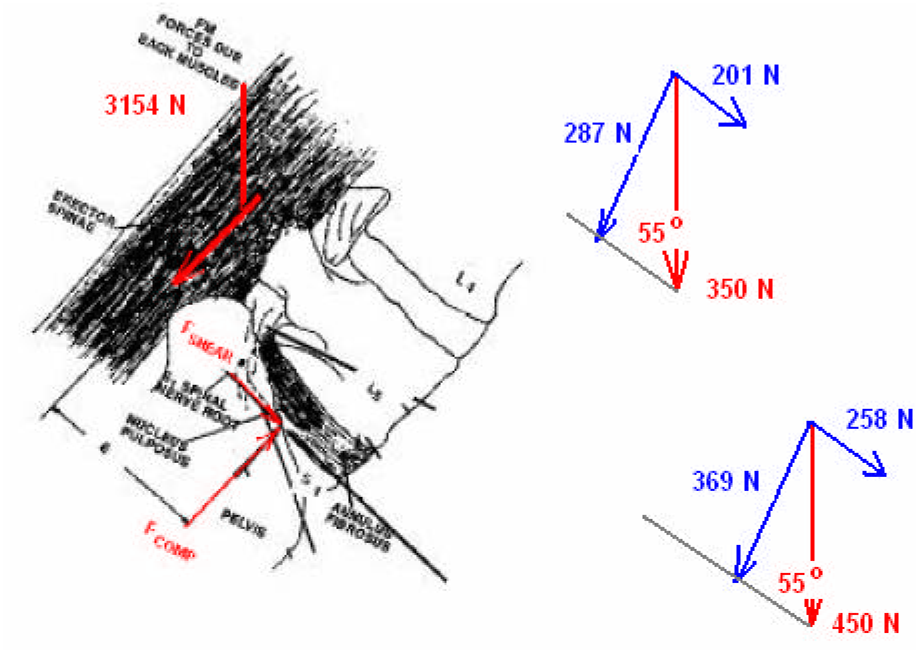
Taking a moment around L5/S1 joint and equating it to Zero, we get

$\sum M = 0$: F_M*6.5 – 350*20-450*30 = 0

Or, F_M= (350*20+450*30)/6.5 = 3154 N (downward)

That is the muscle has to produce a 3154 N force to maintain static equilibrium.

Observe that for this example the muscle force is approximately 7 times the force at hand. Also observe that when while picking up the same load if the upper body leans forward, F_M (back muscle force) will increase as the 0moment arms of the forces will increase.



To find the spine compressive and spine shear forces, we resolve the body weight and force in hand into X and Y components, where Y direction being parallel to the spine axis.

$F_{BWx} = F_{BW} \cos(\alpha) = 350 \cos(55) = 201$ N
$F_{BWy} = F_{BW} \sin(\alpha) = 350 \sin(55) = 287$ N

$F_{Lx} = F_L \cos(\alpha) = 450 \cos(55) = 258$ N
$F_{Ly} = F_L \sin(\alpha) = 450 \sin(55) = 369$ N

Then, equating forces in Y directions,
$\sum F_y = 0$: $F_{COMP} = 3154+287+369 = 3810$ N

Similarly, equating forces in X directions,
$\sum F_x = 0$: $F_{SHEAR} = 201+258 = 459$ N

Thus the spine compressive force is about 3810/450=8.5 times the load in hand.

Epidemiological studies have shown that higher occupational spine compressive force is strongly correlated to incidence of back pain.