# Example Problems on Static Equilibrium

**Example 1.** Suppose one truck is parked on a bridge as shown in **Figure 1**. The truck weighs 1000 lb which is acting through its center of gravity (CG). The bridge weighs 200 lbs per feet, which is uniformly distributed. We can assume the bridge is rigid. We want to know what will be the reaction forces at the supports of bridge at the two ends.

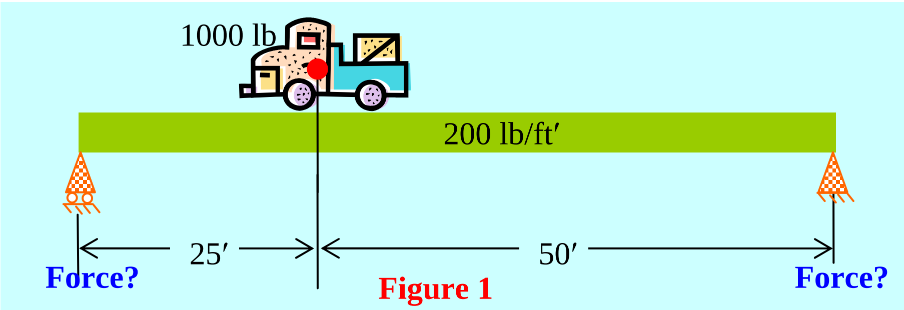



Figure 1

**Step 1: Draw Free Body Diagram (FBD)**

Total weight of the bridge = 200 lb/ft *75ft = 15,000 lb. Since the weight is uniformly distributed; it is acting through the middle of the length of the bridge, ie. 75/2=37.5 ft. from left support. R1 & R2 are the unknown reaction forces at the support. FBD for the bridge is drawn (**Figure 2)** with all the known and unknown forces and the distances between the forces.

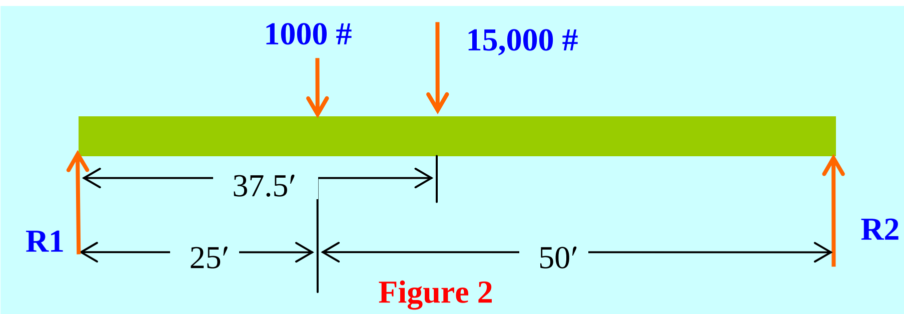



Figure 2

**Step 2: Apply Equilibrium Conditions ($\Sigma F_x=0$; $\Sigma F_y=0$;& $\Sigma M=0$):**

$\Sigma F_x=0$: Produces nothing as no force in x direction

$\Sigma F_y=0$: 1000+15,000-R1-R2 = 0, or R1+R2 = 16,000 ………….. (1)

$\Sigma M=0$:

If we take moment of forces about the left support, then the forces 1000# and 15,000# will produce a clockwise moment (positive). The force R2 will produce a counter-clockwise (negative) moment.

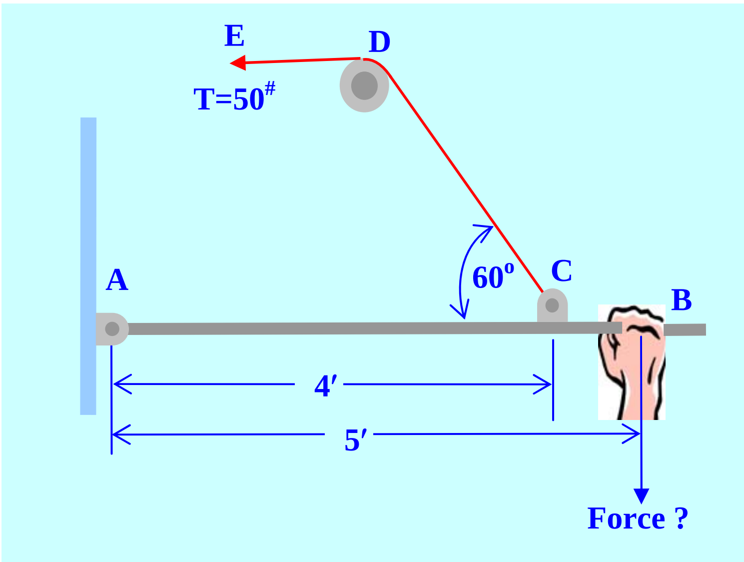
Thus, $\Sigma M=0$: 1000*25+15,000*37.5-R2*75 = 0

Or, R2 = (1000*25+15,000*37.5)/75

Or, **R2 = 7833 lb**

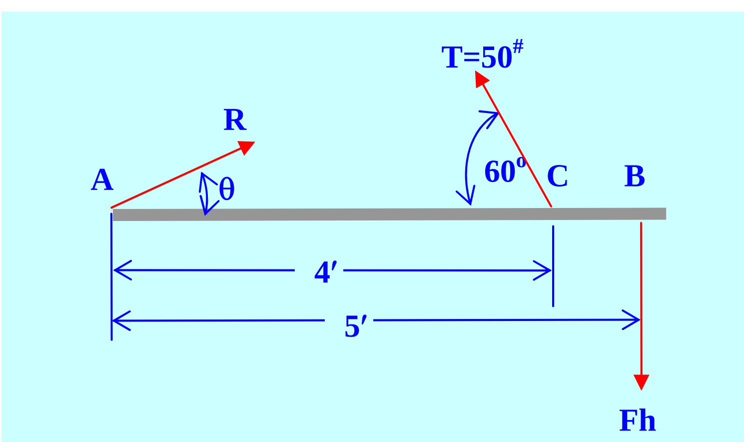
Putting value of R2 in (1): **R1** = 16,000-7833.3 = **8167 lb**

**Example 2.** Suppose rod AB is hinged to a fixed wall. Rope CDE is attached to the rod at C, 4' from the hinge. The rope passes over a pulley D, which is mounted on another fixed wall. The angle ACD is $60^o$. The other end of the rope is connected to a mechanism (not shown), which requires 50 lb force to be actuated. How much downward hand force will be required at point B, 5' from the hinge, to actuate the mechanism? When the hand force is applied, what reaction force will be developed in hinge A? Assume the rod is weightless, rigid and amply strong and the rope is flexible and amply strong.



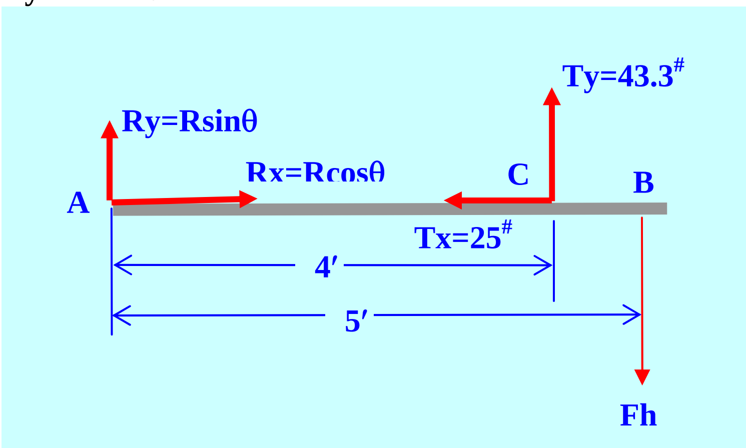

**Step 1: Draw Free Body Diagram (FBD) of the rod**

The tension force in the rope will be of same magnitude on both sides of the pulley, otherwise the pulley will roll. Thus, the force exerted by the rope on the rod will also be 50# acting at $60^o$ angle. At the left hinge, a reaction force will be generated, for which both magnitude (R) and direction (θ) are unknown. The vertically downward hand force (Fh) is also unknown. All these known and unknown forces are shown in FBD (Figure 2).



**Step 2: Replace the inclined forces with X and Y components**

**Tx = Tcos60 = 50 cos60 = 25#**
**Ty = T sin60 = 50 sin60 = 43.3#**
**Rx = R cosθ**
**Ry = R sinθ**




**Step 3: Apply Equilibrium Conditions ($\Sigma F_x=0$; $\Sigma F_y=0$;& $\Sigma M=0$):**

$\Sigma F_x=0$: Rcosθ = 25 ………….(1)
$\Sigma F_y=0$: Rsinθ +43.3=Fh ………….. (2)
ΣM=0: Let us take moment about point A. Since Forces Rx, Ry and Tx passes through the point A, their moment about point A will be zero. Thus

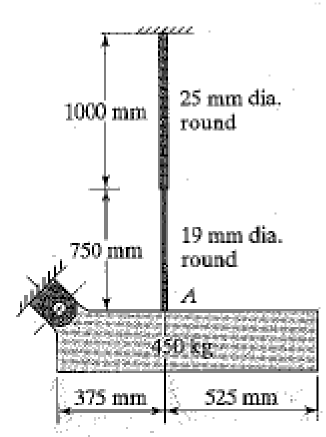
Fh*5-43.3*4 =0, or, Fh = 43.3*4/5 = **34.6#** …….(3)

Putting the value of Fh in (2),
Rsinθ +43.3=34.6
Or, Rsinθ = -8.7……………..(4)
Dividing (4) by (1):
(Rsinθ )/ (Rcosθ) = tanθ = –8.7/25 = –0.346
Or, $\theta = -19.1^o$
Putting value of θ in (1)
Rcos(-19.1) = 25
R = 25/cos (-19.1) = **26.5#**

**Thus the hand force is 34.6# downward and the hinge reaction force is 26.5# at an angle -19.1$^o$ from horizontal.**

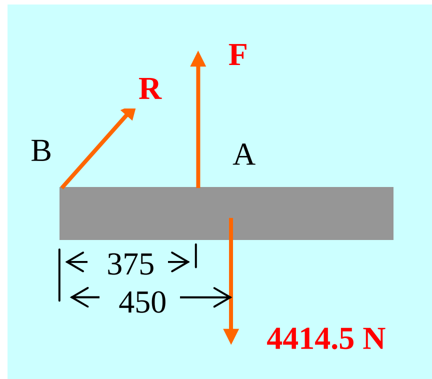
## Example Problem Involving Force & Deformation

**Example 3.** The bottom member in figure below is of uniform cross-section. Its hinge is frictionless. The rods are of steel. Find the distance point A drops upon attachment of the weight. Also find the angle of the bottom member, upon attachment.




The amount of movement of point A will be the sum total of the stretch in the two rods holding the bottom member. To find the stretch, we need to find the axial forces in the rods.
To do that, we draw the **free body diagram (FBD)** of the bottom member. The gravitational force in the bottom member =

450kg*9.81m/sec$^2$ = 4414.5 N, acting through the middle of the member (since, uniform cross-section). The vertically upward force at the rod attachment (F) and the reaction force at the hinge (R) are unknown.



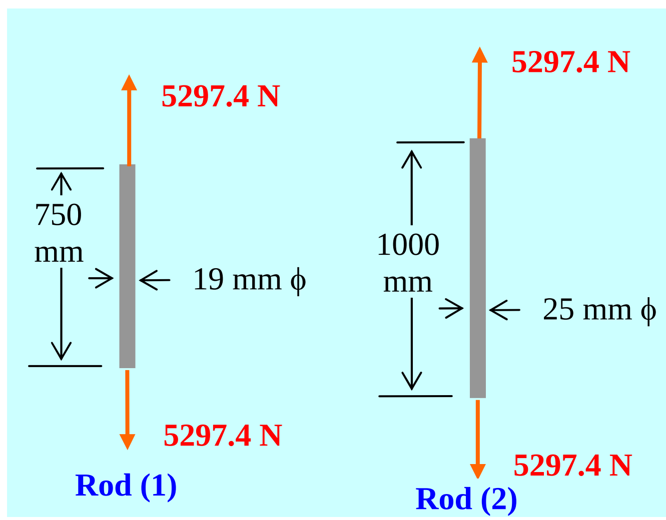

Because the member is in static equilibrium, sum of moments of all forces at any point on the member must be equal to zero. Taking moment around point B:

$\Sigma M_B$ =0: F*375 = 4414.5*450
F = 4414.5*450/375 = 5297.4 N

Thus the force exerted by the rods on the bottom member is 5297.4 N. The same amount of force will be exerted by the bottom member on to the rods. (Newton's third law)

Cross-sectional area of rod (1)
$A_1 = (\pi/4)*19^2 = 283.5\ mm^2$
Cross-sectional area of rod (2)
$A_1 = (\pi/4)*25^2 = 490.9\ mm^2$

Modulus of elasticity for steel = E = 206,900 MPa.

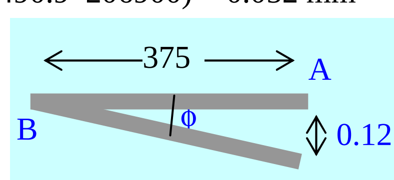



Then the axial deformation of rod (1) = (PL)/AE = (5297.4*750)/(283.5*206900) = 0.068 mm, and the axial deformation of rod (2) = (PL)/AE = (5297.4*1000)/(490.9*206900) = 0.052 mm

**Thus the point A on the bottom member will move a total of .068+.052 = 0.12 mm downward.**

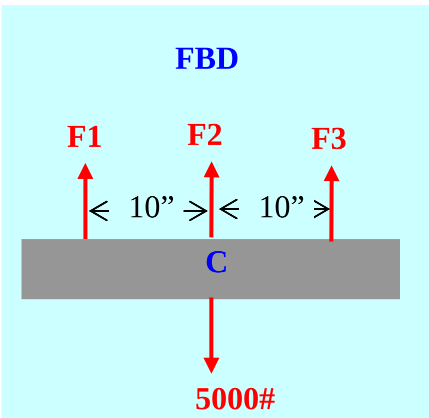
This movement will cause a rotation of the bottom member around the hinge.

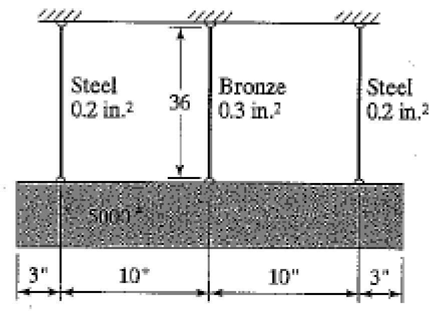
**The angle $\phi$ =** 0.12/375 rad = (0.12/375)*(180/$\pi$) = **0.018°.**



## Example Problem Statically Indeterminate Type

**Example 4.** Three vertical rods of equal length are affixed at the ceiling at one end, one 5000# weight at the other end as shown. The two outer rods have cross-sectional area 0.2 $in^2$ and are made up of steel. The center rod is made up of bronze and has a cross-sectional area 0.3 $in^2$. Find the forces in each rod. Assume the weight is rigid. E for steel = 30,000,000 psi, E for bronze = 15,000,000 psi.




Taking moment about C, $\Sigma M_C = 0$: F1*10-F3*10 = 0 or F1=F3 (This can also be predicted from Symmetry of the bottom weight)

$\Sigma Fy=0$: F1+F2+F3 = 5000; or 2F1+F2 = 5000 …….(1)

$\Sigma Fx=0$, will not give any equation. Thus, using static equilibrium condition we cannot find the forces.

Let us consider the axial deformations of rods.

For rod#1: $\delta 1$= (F1*L1)/(A1*E1)

For rod#2: $\delta 2$= (F2*L2)/(A1*E1)

From symmetry we can guess that the weight will remain horizontal. To keep the weight horizontal, $\delta 1 = \delta 2 = \delta 3$.

That is, (F1*L1)/(A1*E1) = (F2*L2)/(A2*E2)

As L1 = L2, F1 = F2*(A1*E1)/(A2*E2)

Or, F1 = F2* (0.2*30,000,000)/(.3*15,000,000)

Or, F1 = 1.333F2 ……………(2)

Now using (1) & (2): 2*1.333F2+F2=5000

Or, F2 = 5000/3.6666 = 1364 #

Then from (1) F1 = (5000-1364)/2 = 1818 # = F3

**Thus force in steel rods = 1818 lb and force in bronze rod = 1364 lb**