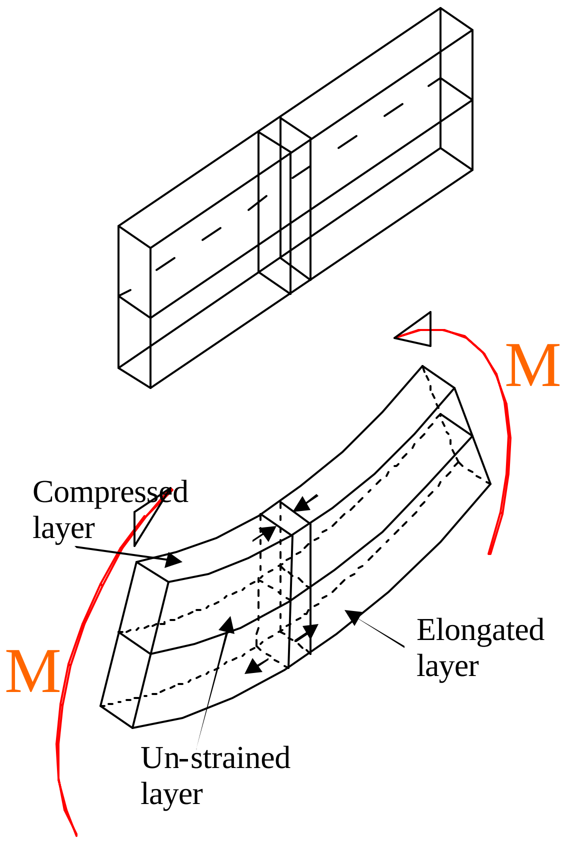
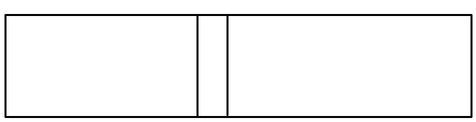
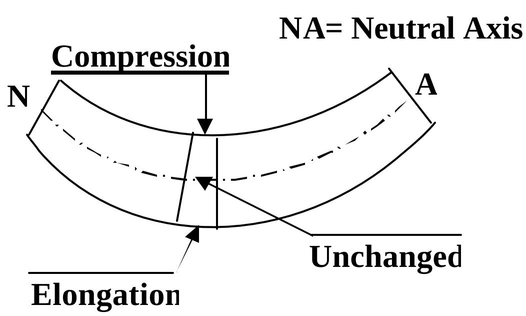
BENDING OF BEAM







Two Dimensional View

When a beam is loaded under pure moment M, it can be shown that the beam will bend in a circular arc.

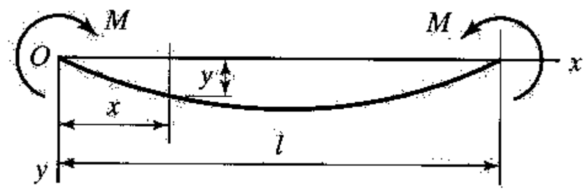
If we assume that plane cross-sections will remain plane after bending, then to form the circular arc, the top layers of the beam have to shorten in length (compressive strain) and the bottom layers have to elongate in length (tensile strain) to produce the curvature. The compression amount will gradually diminish as we go down from the top layer, eventually changing from compression to tension, which will then gradually increase as we reach the bottom layer.

Thus, in this type of loading, the top layer will have maximum compressive strain, the bottom layer will have maximum tensile strain and there will be a middle layer where the length of the layer will remain unchanged and hence no normal strain. This layer is known as Neutral Layer, and in 2D representation, it is known as Neutral Axis (NA).

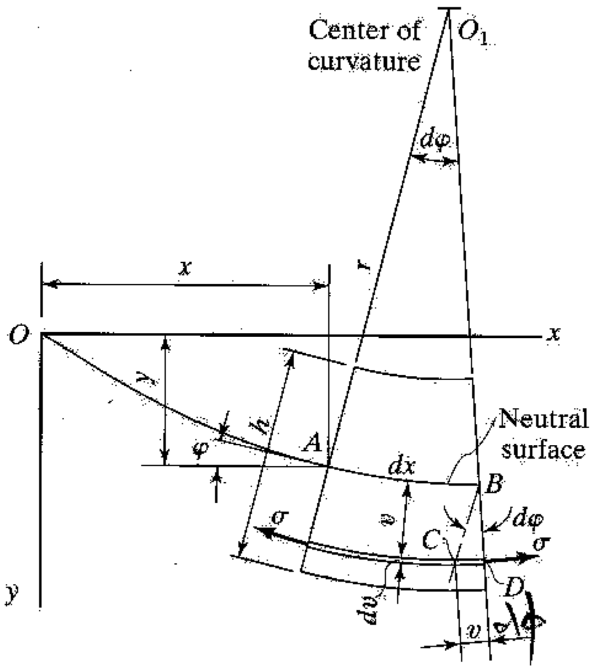
Because the beam is made of elastic material, compressive and tensile strains will also give rise to compressive and tensile stresses (stress and strain is proportional – Hook's Law), respectively.

More the applied moment load more is the curvature, which will produce more strains and thus more stresses.

Our objective is to estimate the stress from bending.



(a)



(b)

We can determine the bending strain and stress from the geometry of bending.

Let us take a small cross section of width dx, at a distance x from the left edge of the beam. After the beam is bent, let the section dx, subtends an angle dϕ at the center of curvature with a radius of curvature r at NA. Then,

$$rd\phi = dx$$

$$\text{or, } \frac{df}{dx} = \frac{1}{r}$$

Let us consider an arbitrary layer at a distance v from the NA. If we draw a line BC parallel to AO_1, then the angle CBD = df. The elongation of this layer = CD= v.dϕ. The original length of this layer was dx.

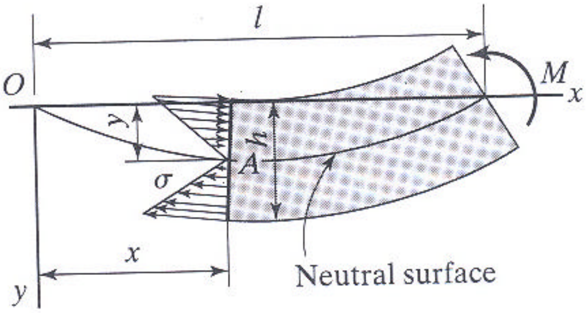
Hence the strain $e = \frac{vdf}{dx} = \frac{v}{r}$ …………..(1)

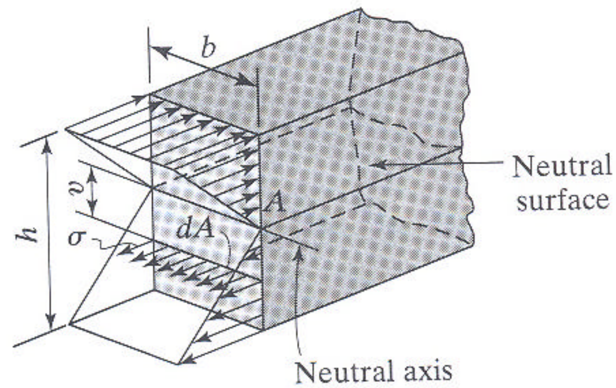
Within elastic limit $stress\ s = Ee = \frac{E}{r}v$ ……(2)

where E = elastic constant.

Since r is fixed for a loading condition, and E is also a constant, then the stress will be proportional to the distance v of any layer from the neutral axis. If we know the radius of curvature due to bending, we can find the bending stress and bending strain using these formulas.

We can now apply the **static equilibrium condition** to one half of the beam and can determine the relationship between the stress and the applied bending moment.





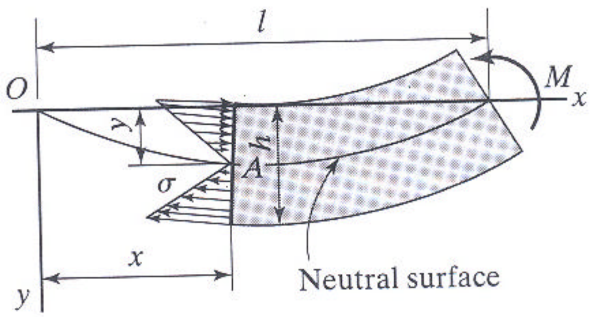
Let dA is an elemental area at a distance v from the neutral plane. The force acting on the element dA is the area multiplied by the bending stress, i.e. σ∗dA = EvdA/r. The total force can be obtained by integration of all forces acting on the face.

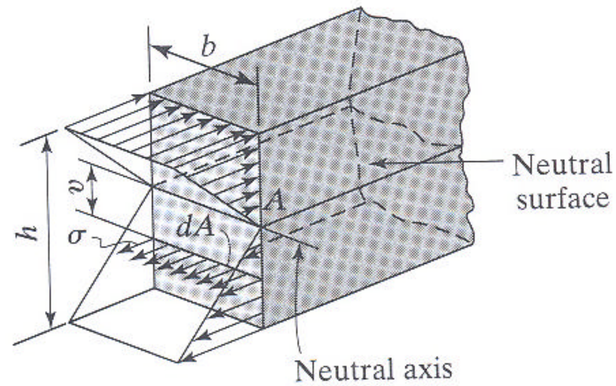
$$Equating\ forces\ in\ X-direction \quad \sum F_x = 0:$$

$$\int s\ dA = \int \frac{E}{r} v dA = \frac{E}{r} \int v dA = 0$$

$$As\ \frac{E}{r} \neq 0 \Rightarrow \int v dA = 0$$

This can only happen if the NA passes through the CG of the cross section. Thus NA must pass through the CG of the beam cross section.





Now if we equate sum total of moments due to bending stress with the applied moment M, then from

$$\sum M = 0:$$

$$M = \int v s \, dA = \int \frac{E}{r} v^2 dA = \frac{E}{r} \int v^2 dA = \frac{E}{r} I = \frac{s}{v} I$$

$$I = Moment\,of\ Inertia\ about\ NA = \int v^2 dA$$

$$s = \frac{M}{I} v \ldots\ldots(3)$$

$$At\ the\ farthest\ layer\ from\ NA,\ v = c,\ \&\ s = s_{\max}$$

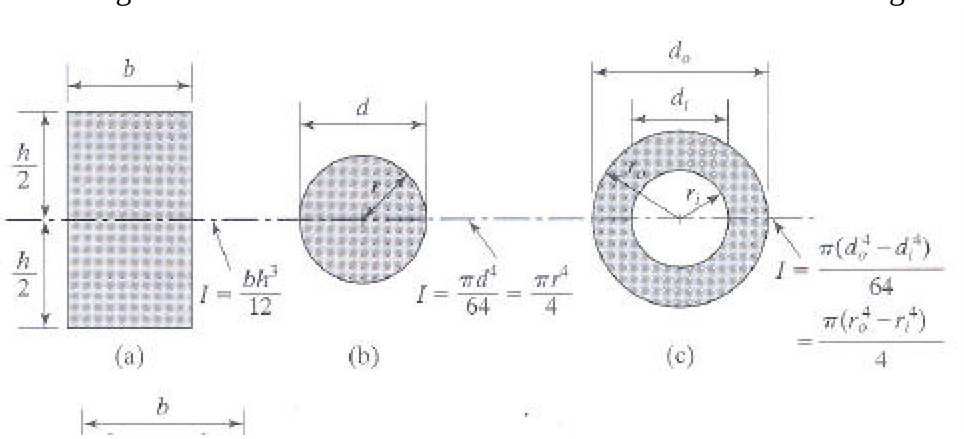
$$Thus,\ s_{\max} = \frac{M}{I} c \ldots\ldots(4)$$

Using equation (3) we can find bending stress at any layer at a distance v from the neutral axis.

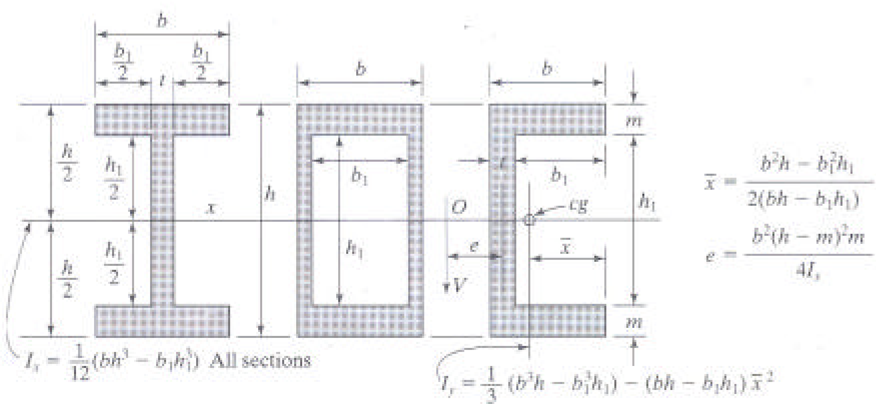
Equation (4) is the formula for maximum bending stress, which will occur at the furthest layer from the NA, where $c = v_{max}$.

MOMENT OF INERTIA (I)

I, for rectangular and circular sections about their NA can be found using following formulae:

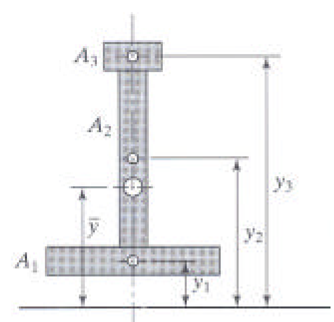


I for I-sections, Box sections and channel sections can be found using following formulae:

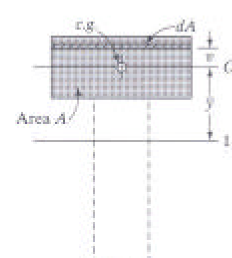


NA passes through the **Center of gravity (CG)** of the beam cross section. For rectangular or circular cross-section of the beam, CG is at the geometric center of the section.

For a composite section, the **location of the CG** can be determined by the following formula,



$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + ...}{A_1 + A_2 + A_3 + ...}$$

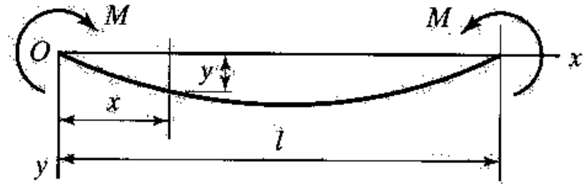


Transfer of axis for Moment of Inertia

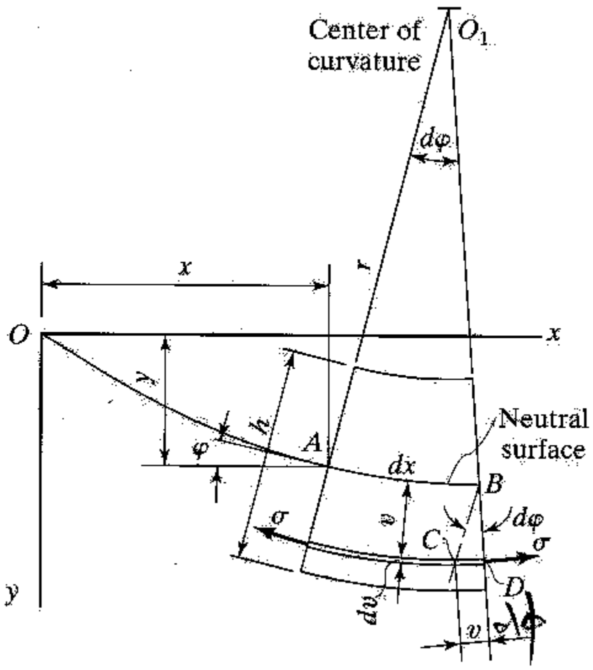
This formula is used to find MI of a T or other sections, whose NA or CG is not located at the geometric symmetrically.

$$I_1 = I_0 + A\bar{y}^2$$

ADDITIONAL BEAM EQUATION



(a)



(b)

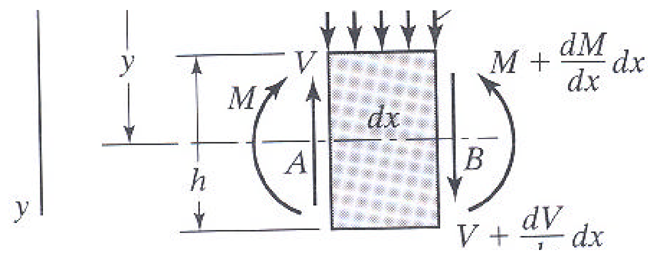
Slope of the beam at x = ϕ,

$$f = \frac{dy}{dx} \;\&\; \frac{df}{dx} = \frac{d^2y}{dx^2}$$

$$\frac{df}{dx} = -\frac{1}{r} = \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = -\frac{M}{EI}$$

fundamental beam equation

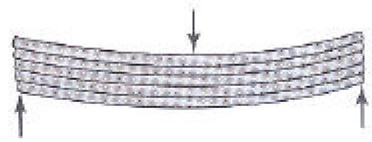


When there are applied forces and moments

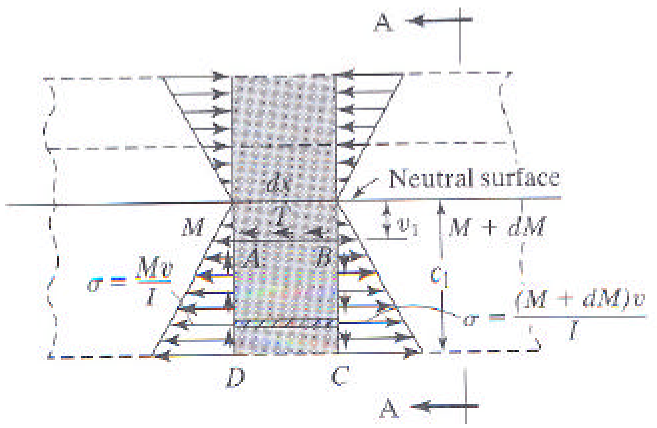
$$M + \frac{dM}{dx}dx - \left(V + \frac{dV}{dx}dx\right)dx - wdx\frac{dx}{2} - M = 0$$

$$\frac{dM}{dx} = V$$

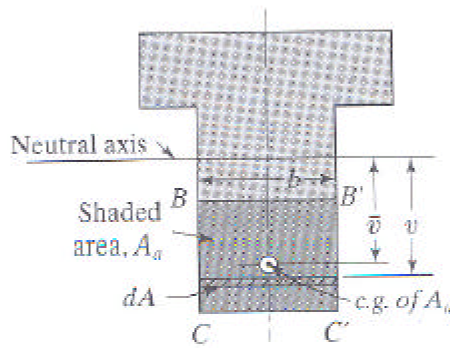
TRANSVERSE SHEAR STRESS



(a) Sliding of laminated strips under transverse shear loads



(b) Side view of beam



(c) Section A–A

The equilibrium equation for horizontal forces for $ABCD$ is then

$$\tau b\, dx + \int_{v_1}^{c_1} \frac{Mv dA}{I} = \int_{v_1}^{c_1} \frac{(M + dM)v dA}{I}$$

$$\tau = \frac{1}{b} \int_{v_1}^{c_1} \frac{dM}{dx} \frac{v dA}{I} = \frac{V}{Ib} \int_{v_1}^{c_1} v dA$$

$$t = \frac{V}{Ib} \bar{v} A_a = \frac{VQ}{Ib}$$

Max. transverse shear stress, τ_{max} always occurs at NA (because Q is max at NA)

Max Transverse shear stresses ($t_{\mathbf{max}}$):

Solid rectangular cross-section	$t_{\max} = \frac{3}{2}\frac{V}{A}$	A = area of the cross-section = b.d
Solid circular cross-section	$t_{\max} = \frac{4}{3}\frac{V}{A}$	A = area of the cross-section = $\frac{p}{4}d^2$
Circular cross section with thin wall*	$t_{\max} = 2\frac{V}{A}$	A = area of the cross-section $= \frac{p}{4}\left(d_o^2 - d_i^2\right)$
I cross-section	$t_{\max} = \frac{V}{A}$	A = t.d t= thickness of the web, and d= total depth of the I-beam

When finding transverse shear stress (τ) for a composite section, the following formula can be used:

$$t = \frac{V}{Ib}\sum \bar{v}A_a = \frac{V}{Ib}\left(\bar{v}_1 A_{a1} + \bar{v}_2 A_{a2} \ldots\right)$$