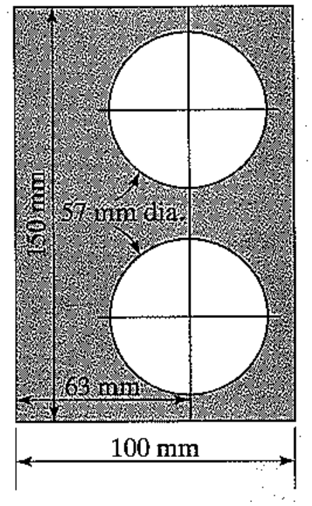
MET 301: Homework#2

1. Find the value of the moment of inertia about the left edge in Fig.
   Ans. $I = 28{,}708{,}000\ \text{mm}^4$.



Rectangle:

$$I_0 = \frac{bd^3}{12} = \frac{150\times100^3}{12} = 12{,}500{,}000\ mm^4$$

$$A = 150\times100 = 15{,}000\ mm^2$$

$$\bar{y} = 50\ mm$$

$$I_1 = I_0 + A\bar{y}^2 = 12{,}500{,}000 + 15{,}000\times50^2 = 50{,}000{,}000\ mm^4$$

One circle:

$$I_0 = \frac{\pi d^4}{64} = \frac{\pi\times57^4}{64} = 518{,}166.5\ mm^4$$

$$A = \frac{\pi\times57^2}{4} = 2551.8\ mm^2$$

$$\bar{y} = 63\ mm$$

$$I_1 = I_0 + A\bar{y}^2 = 518{,}166.5 + 2551.8\times63^2 = 10{,}646{,}096.5\ mm^4$$

MI of the whole section

= 50,000,000 - 2$\times$10,646,096.5= **28,707,807 mm^4**

2. A steel saw blade 1.25 mm thick is bent into an arc of a circle of 600 mm radius. Find the bending stress.
Ans. σ = 215 MPa.

Solution:

Bending radius of the saw blade: r = 600+1.25/2 = 600.625 mm
Bending stress σ = E*c/r = 206900*.625/600.625 = **215 Mpa.**

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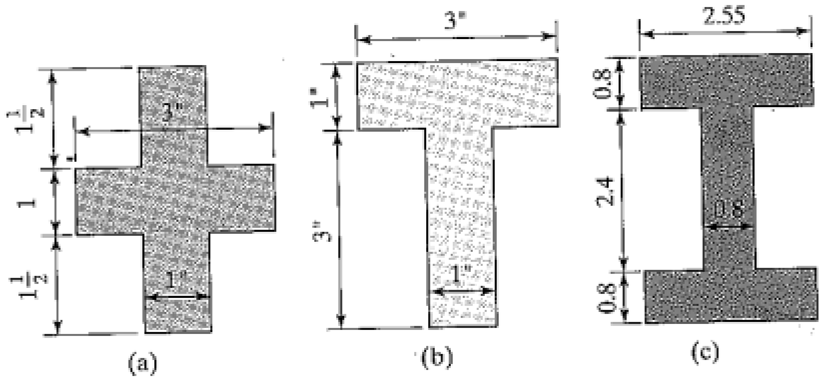
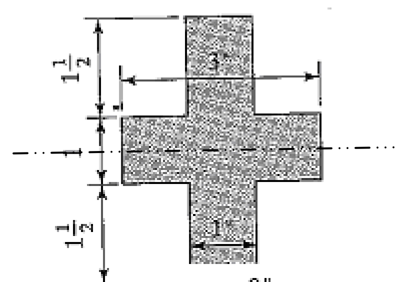
3. Three beams have the equal cross-sectional areas shown in Fig. 1-118. Find the stress in each beam for a bending moment of 30,000 in. lb.
Ans. (b) $\sigma = 8820$ psi.



Solution:

Max stress from bending moment $\sigma = \dfrac{Mc}{I}$



(a) $I = \dfrac{1\times 4^3}{12} + \dfrac{2\times 1^3}{12} = 5.5\,in^4$

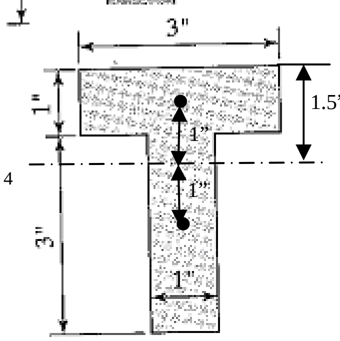
Stress $\boxed{\sigma = \dfrac{Mc}{I} = \dfrac{30{,}000\times 2}{5.5} = 10{,}909\ psi}$

(b) Area $A = 3*1+3*1 = 6\ in^2$

Measuring from the top: $6\bar{y} = 3\times 0.5 + 3\times 2.5;\qquad \bar{y} = 1.5\,in$

Neutral axis is passing at a distance of 1.5 in from the top.

Moment of inertia (apply parallel axis theorem):

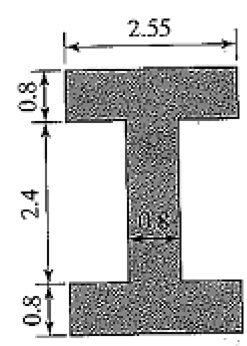


For the Flange $I = I_0 + A\bar{y}^2 = \dfrac{3\times 1^3}{12} + 3\times 1^2 = 3.25\,in^4$

For the web $I = I_0 + A\bar{y}^2 = \dfrac{1\times 3^3}{12} + 3\times 1^2 = 5.25\,in^4$

Total I = 3.25+5.25 = 8.5 in^4

Stress $\boxed{\sigma = \dfrac{Mc}{I} = \dfrac{30{,}000\times 2.5}{8.5} = 8{,}824\ psi}$



(c) Moment of Inertia $I = \dfrac{2.55\times 4^3}{12} - \dfrac{1.75\times 2.4^3}{12} = 11.584\,in^4$

Stress $\boxed{\sigma = \dfrac{Mc}{I} = \dfrac{30{,}000\times 2}{11.584} = 5{,}180\ psi}$

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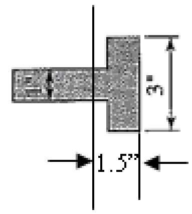
4. Find the values and plot the distribution of stress over the cross section of the upright. Locate the point of zero stress.
Ans. $\sigma_t =$ 13,720 psi

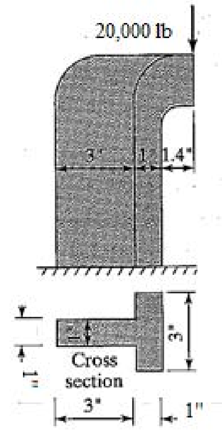
Solution:

Area $A = 3*1+3*1 = 6\ in^2$

Measuring from the top of the flange:

$$\bar{y} = \frac{3\times0.5+3\times2.5}{6} = 1.5 in$$





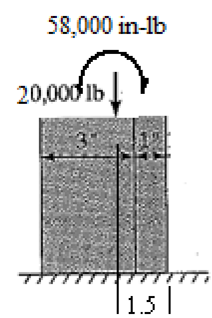
Moment of inertia (applying parallel axis theorem):

For the Flange $I = I_0 + A\bar{y}^2 = \frac{3\times1^3}{12} + 3\times1^2 = 3.25 in^4$

For the web $I = I_0 + A\bar{y}^2 = \frac{1\times3^3}{12} + 3\times1^2 = 5.25 in^4$

Total I = 3.25+5.25 = 8.5 in^4

The upright is subjected compressive force P = 20,000 lb
and a moment M = 20,000*(1.4+1.5)
= 20,000*2.9 = 58,000 in-lb



Max stress at the web =

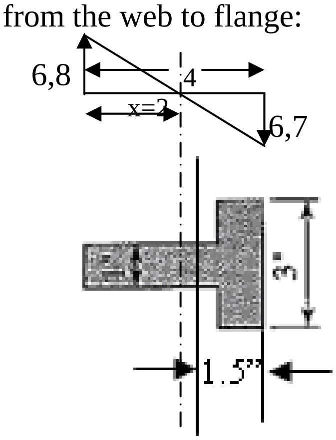
$$\sigma = \frac{Mc}{I} - \frac{P}{A} = \frac{58,000\times2.5}{8.5} - \frac{20,000}{6}$$
$$= 17059 - 3333 = 13,725\ psi\ (tensile)$$

Max stress at the flange =

$$\sigma = -\frac{Mc}{I} - \frac{P}{A} = -\frac{58,000\times1.5}{8.5} - \frac{20,000}{6}$$
$$= -10,235 - 3333 = -13,569\ psi\ (compressive)$$

The stress is changing from tensile to compressive uniformly from the web to flange:

Interpolating: $$x = \frac{4}{(13,725+13569)}\times13725 = 2.01 in$$



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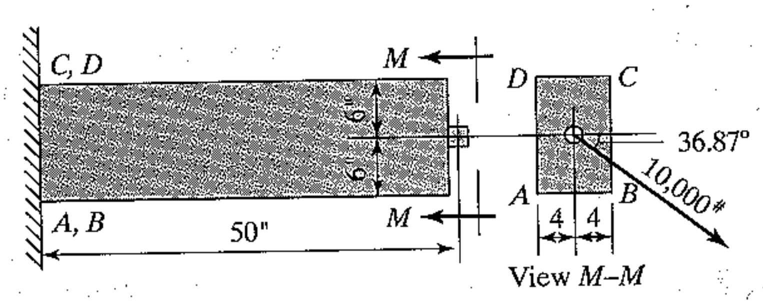
5. The inclined load shown in view *M—M*, Fig. 1-91, is applied to the pin at the end of the beam. Resolve load into horizontal and vertical components and find the bending stress due to each. Add algebraically to get stress at points *A*, *B*, *C*, and *D*.
Ans. At *A*, $\sigma = 1562.5$ psi.

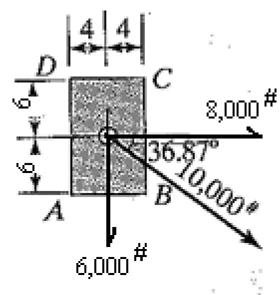


Horizontal:

F_H=10,000 cos 36.87^0 = 8000 lb, I_H = 12*8^3/12 = 512 in^4

Bending stress $\sigma_A = \sigma_D = \frac{Mc}{I} = \frac{8000 \times 50 \times 4}{512} = 3125\ psi$;

$\sigma_C = \sigma_B$ = - 3125 psi



Vertical:

F_V =10,000 sin 36.87^0 = 6000 lb, I_V= 8*12^3/12 = 1152 in^4

Bending stress $\sigma_C = \sigma_D = \frac{Mc}{I} = \frac{6000 \times 50 \times 6}{1152} = 1562.5\ psi$;

$\sigma_A = \sigma_B$ = -1562.5 psi

Summing stresses at four corners algebraically:

σ_A= 3125 -1562.5 = 1562.5 psi (tensile)

σ_B= -3125 -1562.5 = -4867.5 psi (compressive)

σ_C= -3125 +1562.5 = -1562.5 psi (compressive)

σ_D= 3125 +1562.5 = 4867.5 psi (tensile)