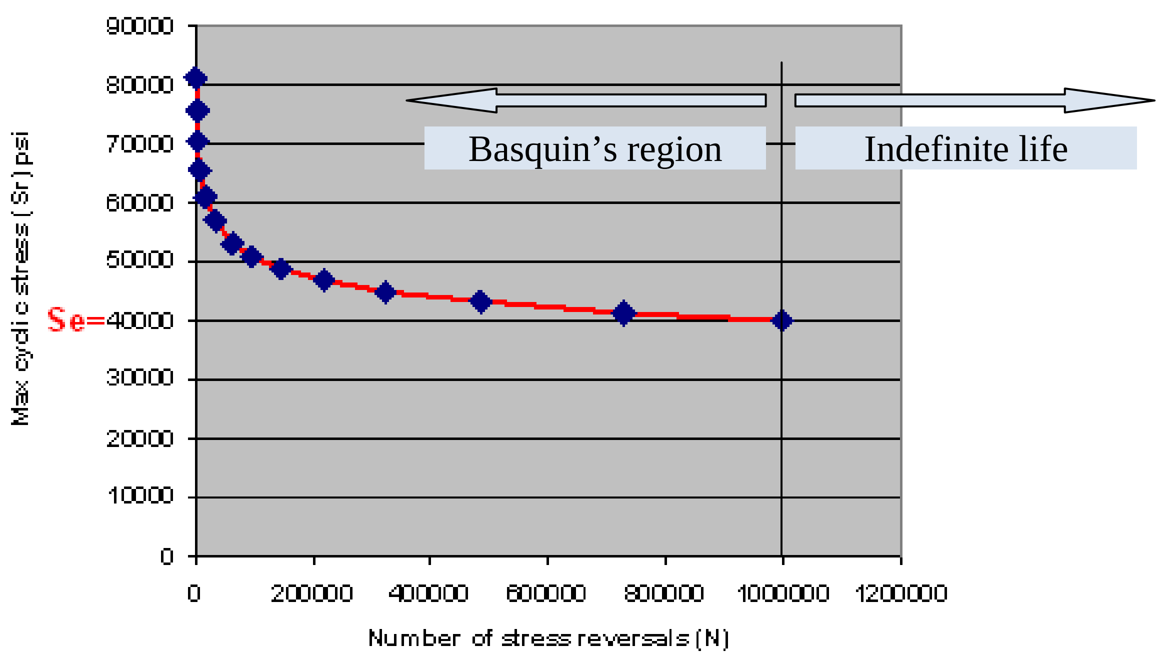
Design for finite life

According to the S-N curve shown below, if the reversing stress (S_r), in a part is kept below the endurance limit (S_e), the part should last infinite number of stress reversals. That is to say that the part will last forever. However, if the part does not need to last forever, we can use slightly higher reversing stress (S_r) to design the part for a finite life and that way save some material in part design. For example, the rotating parts in a booster rocket may be designed to perform for only several thousands of rotations, because the booster will be discarded after few hours of the flight. Obviously, in such a situation, we can use a higher value of reversing stress (S_r) to support the desired number of rotations, and don't have to design for indefinite life. For example, from the S-N curve shown, if the design has to survive up to 100,000 stress reversals, then, the part can be loaded up to a reversing stress level of $S_r \approx 50{,}000$ psi. For indefinite life, S_r is limited to Se= 40,00 psi.



A typical S-N curve

Basquin proposed a mathematical equation to represent the part of the S-N curve for the region $N < 10^6$.

Basquin's Equation: $S_r = AN^B$ ....(1)

S_r is a reversing stress, will give a fatigue life of N cycles ($N<10^6$), and A & B are two material constants.

$$\textit{Taking}\ \log\ \textit{of}\ (1): \quad \log(S_r) = \log A + B\log(N)........(2)$$

For a small number of stress reversals, N = 1000, the range stress, Sr can be approximated to 90% of the ultimate strength of the material:

Putting this pair of values in (2), we get

$$\log(0.9S_u) = \log A + B\log(1000)$$

$$or,\ \log(0.9S_u) = \log A + 3B..........................(3)$$

We also know that Sr = Se, for N = 10^6. Putting values in (2) we get

$$\log(S_e) = \log A + B\log(1000{,}000)$$

$$or,\ \log(S_e) = \log A + 6B..........................(4)$$

Subtracting (4)-(3) and simplifying: $B = \dfrac{\log(S_e) - \log(0.9S_u)}{3}$

From (4): $A = \dfrac{S_e}{10^{6B}}$

Thus from the known values of ultimate strength (Su) and the endurance strength (Se) of the part material, we can determine the material constants A and B of Basquin's equation.

Once the constants A & B are known, then we can determine the maximum completely reversing stress Sr for any value of N (less than 10^6) using Basquin's equation $S_r = AN^B$. Alternatively, for a given completely reversing stress Sr greater than Se, we can determine the number of stress reversals (N) after which fatigue failure will occur:

$$N = \left(\frac{S_r}{A}\right)^{\frac{1}{B}}$$

Example 2-5

Suppose that a material with σ_u=90,000 psi and σ_e=40,000 psi, is to be subjected to a completely reversing stress of σ_r=45,000 psi. Use Basquin's equation to determine the expected number of cycles to failure for this stress level.

Solution:

(i) Find A & B of Basquin's Equation using σ_u and σ_e.

$$B = \frac{\log(S_e) - \log(0.9S_u)}{3}$$
$$= \frac{\log(40{,}000) - \log(0.9*90{,}000)}{3}$$
$$= \frac{4.602 - 4.9085}{3} = -0.10214$$

$$A = \frac{S_e}{10^{6B}} = \frac{40{,}000}{10^{(6*-0.10214)}} = 164{,}025$$

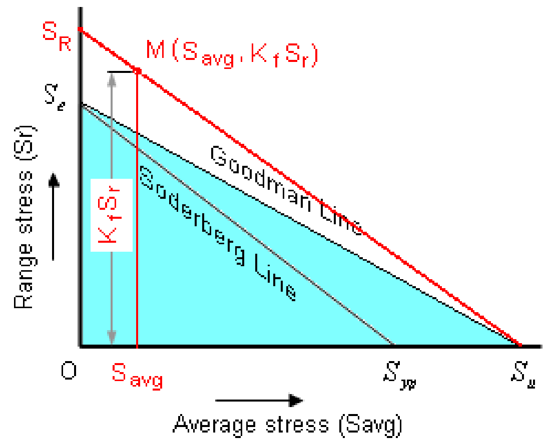
(ii) Apply Basquin's Equation to calculate N

$$N = \left(\frac{S_r}{A}\right)^{\frac{1}{B}} = \left(\frac{45{,}000}{164025}\right)^{\frac{1}{-0.10214}} = 315{,}646$$

Combined steady (S_{avg}) and range stress (S_r)

For a combined steady (S_{avg}) and range (S_r) stress, we first need to determine the equivalent reversing stress (S_R) from the combined stress situation. Once the equivalent range stress (S_R) is determined, then Basquin's equation can be used to obtain the number of reversals N for this equivalent S_R.

The equivalent range stress S_R from the applied combined stress (S_{avg} & K_f S_r) is determined from Goodman line shown (with N_{fs}=1).



We know that a combined stress denoted by a point below the Goodman line (in blue shaded area), will provide indefinite life. For finite life situation, the applied stress point M (S_{avg}, K_fS_r) must be above the Goodman line. To obtain the equivalent reversing stress, we draw a line from S_u on x axis to the point M and extend the line to y axis. The point S_R is the intersection of this line to y axis, which denotes the equivalent range stress. The magnitude of S_R can be easily obtained from the two similar triangles ΔOS_uS_R and $\Delta S_{avg}MS_u$: $\frac{S_R}{S_u} = \frac{K_f S_r}{S_u - S_{avg}}$

$$\boxed{or, S_R = \frac{K_f S_r S_u}{S_u - S_{avg}}}$$

Thus, for designing with finite life of a part ($N<10^6$), the applied S_{avg} & S_r can be converted to an equivalent S_R and then Basquin's equation can be applied.

Miner’s Equation for Finite Life Design

A machine part may be subjected to one combination of steady and reversing stress for a portion of its life, another combination of steady and reversing stress for another portion of its life, and so on. Each combination of S_{avg} & S_r, can be reduced to equivalent completely reversed stresses S_{R1}, S_{R2}, S_{R3} and so on.

Let fatigue life for stress S_{R1}, if applied alone is “N_1” reversals, and let “n_1” be the actual number of reversals at stress level S_{R1}. Then stress S_{R1} has consumed “n_1/N_1” portion of the life of the part. Similarly, let “N_2” be the fatigue life at stress S_{R2}, and let “n_2” be the actual reversals occurred at this stress and thus consumed “n_2/N_2” portion of life. Sum of these proportions constitutes the entire life, or unity:

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + ... = 1$$

The above equation is called Miner’s Equation. Using this equation we can determine the amount of life remaining in a part at any stress level, if we know the past history of its service.

Example 2-6

Suppose that a part made with a material with σ_u=90,000 psi and σ_e=40,000 psi, is to be subjected to a loading cycle with K_f = 1.5 that uses:

320,000 cycles at σ_{avg}=45,000 psi and σ_r=13,641 psi, and

18,600 cycles at σ_{avg}=35,000 psi and σ_r=20,787 psi.

How many cycles remain for the loading condition σ_{avg}=48,000 psi and σ_r=15,000 psi?

(i) Find the equivalent reversing stresses, from the combined steady and reversing stresses:

$$S_{R1} = \frac{K_f S_{r1} S_u}{S_u - S_{avg1}} = \frac{1.5 \times 13641 \times 90{,}000}{90{,}000 - 45{,}000} = 40{,}923\ psi$$

$$S_{R2} = \frac{K_f S_{r2} S_u}{S_u - S_{avg2}} = \frac{1.5 \times 20{,}787 \times 90{,}000}{90{,}000 - 35{,}000} = 51{,}023\ psi$$

$$S_{R3} = \frac{K_f S_{r3} S_u}{S_u - S_{avg3}} = \frac{1.5 \times 15{,}000 \times 90{,}000}{90{,}000 - 48{,}000} = 48{,}214\ psi$$

(ii) Since σ_u=90,000 psi and σ_e=40,000 same as our previous example A = 164,025 and B= -0.10214. Using Basquin's Equation, find fatigue lives at the above three equivalent reversing stresses:

$$N_1 = \left(\frac{S_{R1}}{A}\right)^{\frac{1}{B}} = \left(\frac{40{,}923}{164025}\right)^{\frac{1}{-0.10214}} = 799{,}839$$

$$N_2 = \left(\frac{S_{R2}}{A}\right)^{\frac{1}{B}} = \left(\frac{51{,}023}{164025}\right)^{\frac{1}{-0.10214}} = 92{,}279$$

$$N_3 = \left(\frac{S_{R3}}{A}\right)^{\frac{1}{B}} = \left(\frac{48{,}214}{164025}\right)^{\frac{1}{-0.10214}} = 160{,}648$$

(iii) Apply Miner's equation to find the remaining life n_3:

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1 \Rightarrow \quad \frac{320{,}000}{799{,}839} + \frac{18{,}600}{92{,}279} + \frac{n_3}{160{,}648} = 1$$

$$\Rightarrow \quad \frac{n_3}{160{,}648} = 1 - 0.4 + 0.2015 = .3984 \quad \Rightarrow$$

$$n_3 = 160{,}648 \times .3984 \approx 64{,}000$$

Concluding Remark

1. As you have notices that all the theories about cyclic stress developed here pertain to uniaxial stress. For **biaxial or triaxial stresses**, the stresses may interact differently, and prediction of failure is not straight forward. In case of cyclic stress one more problem arises, that is **the cyclic stresses in different axes may have different frequencies**. Design procedure for such stresses is beyond the scope of this course.

2. Failure with fluctuating stresses that are high enough to cause failure in a few thousand cycles or less is called **low-cycle fatigue**. Plastic yielding at localized areas may be involved. The theory is beyond the scope of this course.