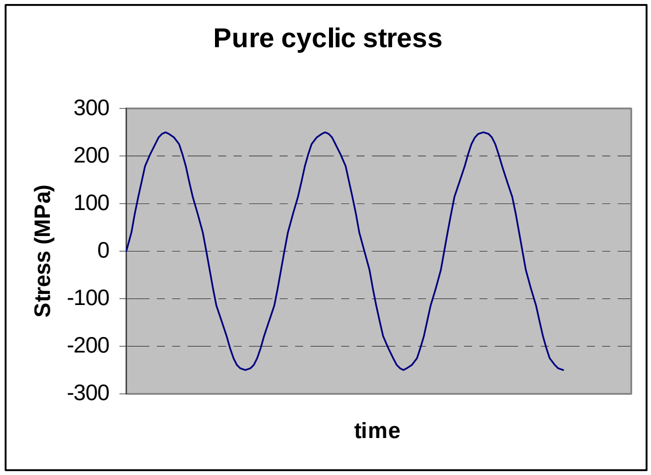
Design for Cyclic Loading

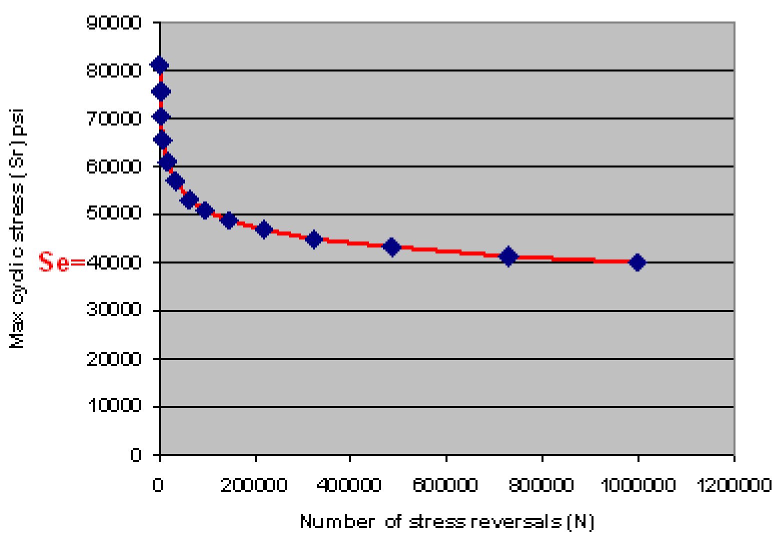
1. Completely reversing cyclic stress and endurance strength

A purely reversing or cyclic stress means when the stress alternates between equal positive and negative peak stresses sinusoidally during each cycle of operation, as shown. In this diagram the stress varies with time between +250 MPa to -250MPa. This kind of cyclic stress is developed in many rotating machine parts that are carrying a constant bending load.



When a part is subjected cyclic stress, also known as range or reversing stress (S_r), it has been observed that the failure of the part occurs after a number of stress reversals (N) even it the magnitude of S_r is below the material's yield strength. Generally, higher the value of S_r, lesser N is needed for failure.

No. of stress reversals (N)	Cyclic stress (Sr) for failure psi
1000	81000
2000	75465
4000	70307
8000	65501
16000	61024
32000	56853
64000	52967
96000	50818
144000	48757
216000	46779
324000	44881
486000	43060
729000	41313
1000000	40000



A typical S-N curve

For a typical material, the table and the graph above (S-N curve) show the relationship between the magnitudes Sr and the number of stress reversals (N) before failure of the part. For example, if the part were subjected to S_r= 81,000 psi, then it would fail after N=1000 stress reversals. If the same part is subjected to S_r = 61,024 psi, then it can survive up to N=16,000 reversals, and so on.

It has been observed that for most of engineering materials, the rate of reduction of Sr becomes negligible near the vicinity of N = 10^6 and the slope of the S-N curve becomes more or less horizontal. For the curve shown above, at N= 10^6, the slope of the curve has become horizontal at Sr=40,000 psi. Because the slope of the above S-N curve is horizontal at N=10^6 reversals, that means if we keep the cyclic stress value less than Sr = 40,000 psi, then the part will not fail no matter how many cycles have been operated, ie survive indefinitely.

The maximum completely reversing cyclic stress that a material can withstand for indefinite (or infinite) number of stress reversals is known as the fatigue strength or endurance strength (Se) of the part material. This is essentially the max completely reversing cyclic stress that a material can withstand for N=10^6 or more, as the curve is horizontal after this point. For the example S-N curve shown above, Se = 40,000 psi.

2. Factors affecting fatigue strength

The failure caused by cyclic stress is called fatigue failure. The fatigue failure originates from a micro-crack (brittle failure) due to stress concentration effect at surface irregularities. Minute irregularities such as grinding scratches, tool marks, inspection stamping, stamped part numbers or surface scales may produce a high value for the stress due to stress concentration and serve as the starting point for the micro crack, which progressively increases until the part breaks into two pieces similar to a brittle fracture. No plastic deformation is observed in the failure surface.

Most commonly, the fatigue failure occurs due to reversing bending stress in rotating machineries, less commonly due to reversing torsional shear stress and rarely from reversing axial stress. As the most highly stressed points are on the outer surface for both bending and torsion, crack originates on the outer surface and progresses inwardly. As a consequence of this, a weak decarburized layer on the outer surface of heat-treated parts often produce low endurance limit.

Since fatigue cracks are due to tensile stress, a residual tensile stress on the surface of the part constitutes a fatigue hazard. Such a residual tensile stress, for example may arise from a cold working operation on the part without stress relieving. Parts that are finished by grinding frequently have a thin surface layer with high residual stress in tension. Such residual stress combined with the tensile stress from loading, may give a resultant stress sufficiently great to cause a fatigue crack to start.

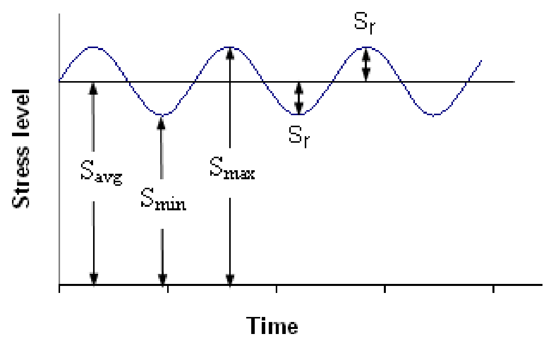
Any residual tensile stress on the surface should be removed, or better still, converted into a layer of compression. Shot blasting, or shot peening operations can induce residual compressive residual stress on surface. Also nitrided and carburised parts have residual compressive stress on the surface. Residual compressive stress on the surface will reduce the tensile stress from the load, and thus helps to improve fatigue characteristics of the part. Sand blasting of the part should be avoided since the scratches serve as stress risers.

The fundamental measure of fatigue performance for a material is the endurance limit from a plain polished specimen. Because the brittle nature of the failure, the endurance limit of a material is closely related to the ultimate tensile strength (S_u) rather than yield strength (S_{yp}). See Figure 2-26 in textbook, which provides the relationships between the endurance limits and the ultimate tensile strengths of steel specimens with different surface finishes.

Typically for wrought steels when the surface is ground and polished Se = 0.5 Su, for machined surface, Se = 0.35 to 0.4 Su and for as-forged or as-rolled surface Se = 0.2 Su. Corrosion of surface from water or acid may reduce the endurance strength to further lower value.

3. Design for fatigue stress

Unlike a pure reversing stress (S_r) discussed above, a machine part may be subjected to a combined steady and reversing stress. Following design procedure handles such combined stress situation. A generalized stress condition, can be defined as combine purely reversing stress (S_r) superimposed on a steady stress (S_{avg}). The following stress-time graph shows this combined reversing and steady stress condition. If the stress is varying between Smax & Smin, then the



Combined cyclic and steady stress

Steady stress = $S_{avg} = \dfrac{S_{\max} + S_{\min}}{2}$

Reversing stress = $S_r = \dfrac{S_{\max} - S_{\min}}{2}$

SODERBERG'S LINE

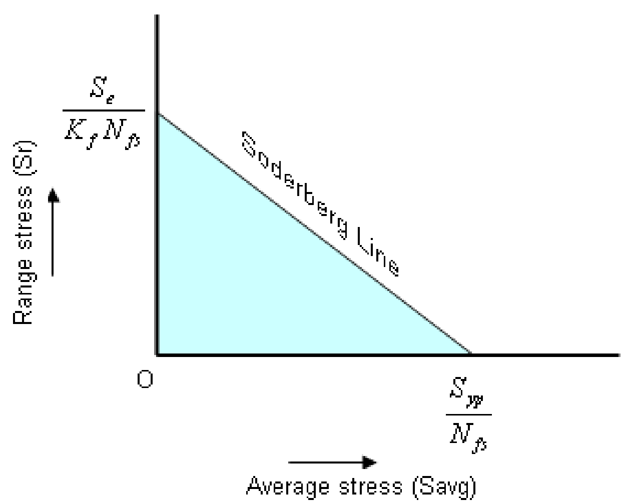
(i) If a part only contains the steady part of the stress S_{avg}, (that is S_r=0) then to prevent failure:

$S_{avg} < S_{yp}/(K^*N_{fs})$, where K= geometric stress concentration factor, and N_{fs}= factor of safety. Usually parts subjected to fatigue loading are made of ductile material, and for steady stress, we learned that the geometric stress concentration factor can be neglected. Thus the limiting condition is:

$$S_{avg} < S_{yp}/N_{fs}$$

Which means that S_{avg} can go up to S_{yp}/N_{fs} when $S_r = 0$

(ii) Similarly, when there is only reversing stress S_r present, then for safe design: $S_r < S_e/(N_{fs}{*}K_f)$, where K_f= is the fatigue stress concentration factor.



Which means S_r can go up to $S_e/(N_{fs}{*}K_f)$, when S_{avg}=0

If we plot steady stress (S_{avg}) along x axis and the range stress (S_r) along y axis, then the two extreme stress conditions (i) & (ii) described above, constitute two point on x and y axis. Soderberg Line is obtained by joining these two points. When in a machine part, both types of stress are present simultaneously, if the stress combination (S_{avg} & S_r) is contained in the blue area defined by the Soderberg line, then the part should be safe. Any stress combination falling above the Soderberg's line would be unsafe.

Using intercept form of the equation of straight line, ie., x/a+y/b=1, the safe design area (blue area) can be defined by:

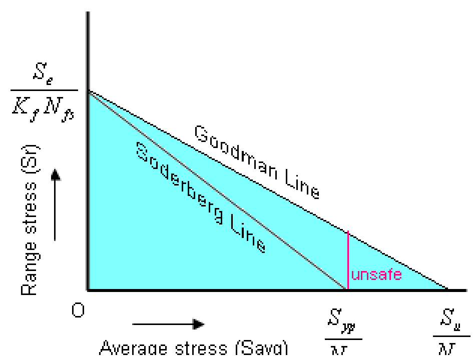
$$\frac{S_{avg}}{\frac{S_{yp}}{N_{fs}}} + \frac{S_r}{\frac{S_e}{K_f N_{fs}}} \le 1; \quad \textit{Multiplying both side by } \frac{S_{yp}}{N_{fs}}$$

$$S_{avg} + S_r K_f \left(\frac{S_{yp}}{S_e}\right) \le \frac{S_{yp}}{N_{fs}} \text{.....................(1)}$$

Equation (1) is called Soderberg Equation for design of a part with combined steady and range stress. Note that, the right hand side of the equation $\frac{S_{yp}}{N_{fs}}$, which is the design limit for normal steady stress S_{avg}. Because of the presence of the range stress S_r, the factor $S_r K_f \left(\frac{S_{yp}}{S_e}\right)$ is added, which is the static equivalent of the range stress S_r.

GOODMAN'S LINE

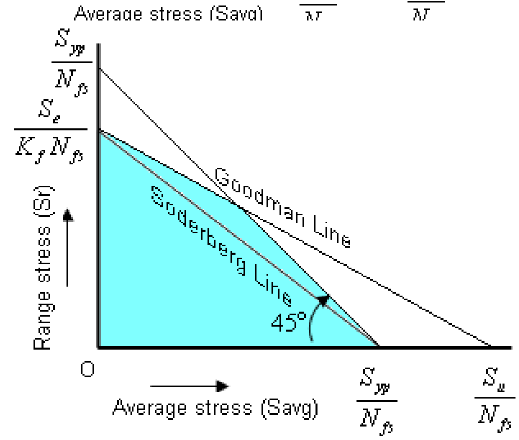
Because of brittle nature of failure, Goodman proposed the safe design stress for steady stress should be extended to S_u/N_{fs} instead of S_{yp}/N_{fs} in Soderberg's equation. This resulted in the safe design space as shown and the resulted in Goodman Design equation:



$$S_{avg} + S_r K_f \left(\frac{S_u}{S_e}\right) \le \frac{S_u}{N_{fs}} \text{.........(2)}$$

Goodman Equation can be obtained from Soderberg equation by replacing S_{yp} by S_u.

However, in the safe area defined by Goodman line, when the magnitude of steady stress S_{avg} becomes more than S_{yp}/N_{fs}, the part may fail from yielding from plastic deformation. The area is shown as unsafe region.



MODIFIED GOODMANS LINE

To eliminate this shortcoming, a line with 45° angle from the S_{yp}/N_{fs} point on the x axis. Mathematically this Modified Goodman space is equivalent to satisfying the following two equations (3) & (4), simultaneously.

$$S_{avg} + S_r K_f \left(\frac{S_u}{S_e}\right) \le \frac{S_u}{N_{fs}} \text{.........(3)}; \quad S_{avg} + S_r K_f \le \frac{S_{yp}}{N_{fs}} \text{................(4)}$$

Example 1: An automobile engine part rotates, and in each rotation stress varies from S_{max}=20,000 psi to S_{min}=1,000 psi. The material has S_u= 80,000 psi, S_{yp} = 60,000 psi, S_e=28,000 psi. Assume K=K_f=1. Find N_{fs}, with (i) **Soderberg's**, (ii**) Goodman's** and (iii) **modified Goodman's equations.**

Solution:

Steady stress = $S_{avg} = \dfrac{S_{max} + S_{min}}{2} = \dfrac{20{,}000 + 1000}{2} = 10{,}500\ psi$

Reversing stress = $S_r = \dfrac{S_{max} - S_{min}}{2} = \dfrac{20{,}000 - 1000}{2} = 9{,}500\ psi$

Soderberg's Equation:

$$N_{fs} = \frac{S_{yp}}{S_{avg} + S_r K_f \left(\dfrac{S_{yp}}{S_e}\right)} = \frac{60{,}000}{10500 + 9500 \times 1 \times \left(\dfrac{60000}{28000}\right)} = 1.94$$

Goodman's Equation:

$$N_{fs} = \frac{S_u}{S_{avg} + S_r K_f \left(\dfrac{S_u}{S_e}\right)} = \frac{80{,}000}{10500 + 9500 \times 1 \times \left(\dfrac{80000}{28000}\right)} = 2.12$$

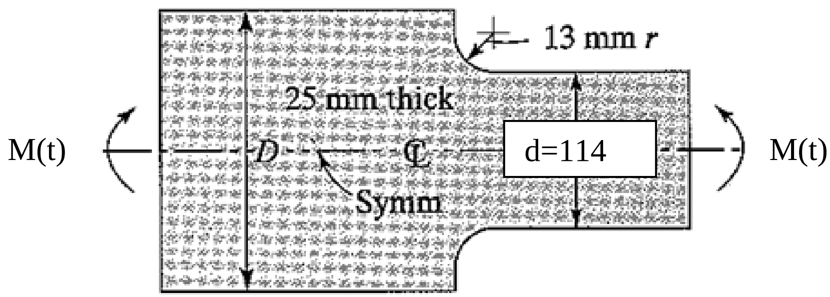
Modified Goodman's Equation:

$$N_{fs1} = \frac{S_u}{S_{avg} + S_r K_f \left(\dfrac{S_u}{S_e}\right)} = \frac{80{,}000}{10500 + 9500 \times 1 \times \left(\dfrac{80000}{28000}\right)} = 2.12$$

$$N_{fs2} = \frac{S_{yp}}{S_{avg} + S_r K_f} = \frac{60{,}000}{10500 + 9500 \times 1} = 6.32$$

$$N_{fs} = 2.12 \ \ (smaller\ one)$$

Example 2: What is the factor of safety using ***Modified Goodman's equation*** if the part is subjected to moment load (*M_t*) varying between 2,250,000 N-mm and 1,250,000 N-mm in each cycle? The geometric stress concentration factor at the base of the radius is *K* = 1.8. The part is made of AISI 1020 steel, with S_{yp} = 350 MPa, S_u= 420 MPa, S_e = 190 MPa, and q = 0.6



From the given M_{max} and M_{min} values, find the M_{avg} and M_r values:

$$M_{avg} = \frac{M_{\max}+M_{\min}}{2} = \frac{2250000+1250000}{2} = 1750000\ N-mm$$

$$M_r = \frac{M_{\max}-M_{\min}}{2} = \frac{2250000-1250000}{2} = 500000\ N-mm$$

To determine the bending stresses:

$$I = \frac{bh^3}{12} = \frac{25\times 114^3}{12} = 3086550\ mm^4; \quad c = \frac{114}{2} = 57\ mm$$

$$S_{avg} = \frac{M_{avg}c}{I} = \frac{1750000\times 57}{3086550} = 32.3\ MPa\ ;$$

$$S_r = \frac{M_r c}{I} = \frac{500000\times 57}{3086550} = 9.2\ MPa$$

The fatigue stress concentration factor (*K_f*):

$$K_f = (K-1)\times q + 1 = (1.8-1)\times .6 + 1 = .8\times .6 + 1 = 1.48$$

Modified Goodman's equation:

$$N_{fs1} = \frac{S_u}{S_{avg} + S_r K_f \left(\frac{S_u}{S_e} \right)} = \frac{420}{32.3 + 9.2 * 1.48 \left(\frac{420}{190} \right)} = \frac{420}{62..399} = 6.7$$

$$N_{fs2} = \frac{S_{yp}}{S_{avg} + S_r K_f} = \frac{350}{32.3 + 9.2 * 1.48} = \frac{350}{45.916} = 7.62$$

$$N_{fs} = 6.7 \ (lesser\ of\ the\ two)$$