ANALYSIS & DESIGN
OF MACHINE ELEMENTS – I
MET 301

SUMMARY OF TOPICS & FORMULAE

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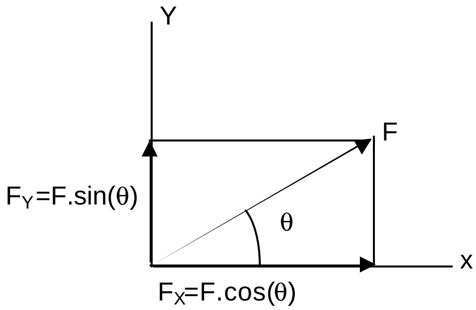
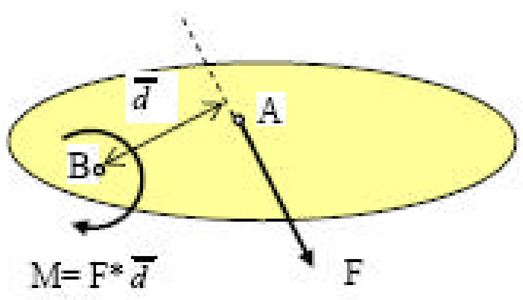
1. RESOLVING A FORCE IN TWO ORTHOGONAL DIRECTIONS



If we know the magnitude of the force F, and its angle (θ) with x-axis, then the component of the force in X and Y directions are:
F_x= F.cosθ,

2. MOMENT OF A FORCE ABOUT A GIVEN POINT



Sign convention:
Clockwise (CW) = positive
Counter clockwise (CCW) = negative

3. STATIC EQUILIBRIUM IN TWO DIMENSIONS

When several forces (and moments) are acting on a body, the body will maintain static equilibrium if the following three equalities are satisfied simultaneously:

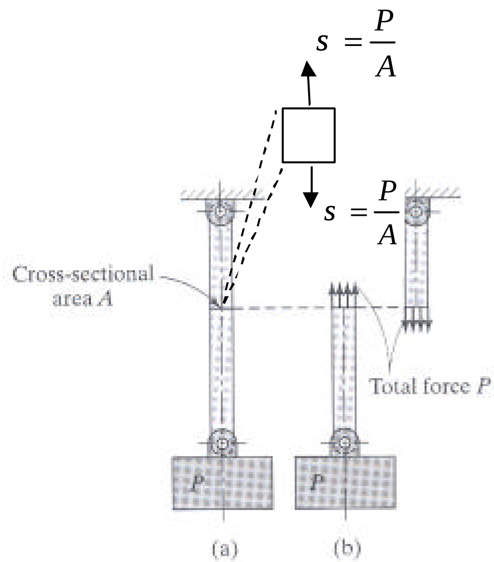
(i) Sum of all forces acting on the body in X direction = 0; i.e., $\Sigma \mathbf{F_x = 0}$

(ii) Sum of all forces acting on the body in Y direction = 0; i.e., $\Sigma \mathbf{F_Y = 0}$

(iii) Sum of moments about any point on the body due to all forces acting on the body = 0; i.e., $\Sigma \mathbf{M = 0}$

These equations are frequently used to find unknown reaction forces and moments due to externally applied known forces and moments.

4. AXIAL TENSION AND COMPRESSION: STRESS, STRAIN & HOOK'S LAW



When a member is axially loaded with a force P, the force acting on any cross-section across the length of the component = P.

The normal stress at any cross section can be obtained by $s = \frac{P}{A}$, where A = cross-sectional area.

Sign conventions:
Tensile stress is positive, compressive stress is negative.

Statically indeterminate problems:
Some times, for axially loaded members, the load shared by each member cannot be determined from static analysis of forces. In those cases, additional deformation relationships can be used to find the load shared.

Normal stress: $s = \frac{P}{A}$, P = Axial load, A = cross sectional area and,

Normal strain: $e = \frac{d}{l}$, δ = elon`gation, L = original length.

Hooks law: Stress (σ) is proportional to strain (ε)

$\sigma = E.\varepsilon$, E = Elastic Modulus or Young's Modulus

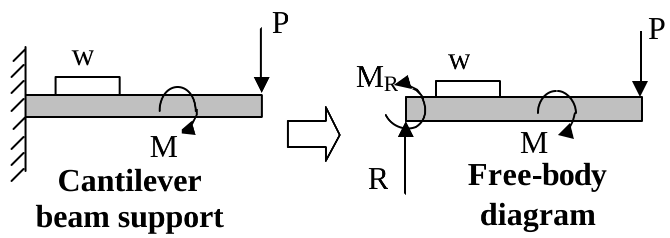
Thus, $\frac{P}{A} = E\frac{d}{L}$.

Rearranging: $d = \frac{PL}{AE}$

5. TRANSVERSE LOADING:
INTERNAL SHEAR FORCE & BENDING MOMENT

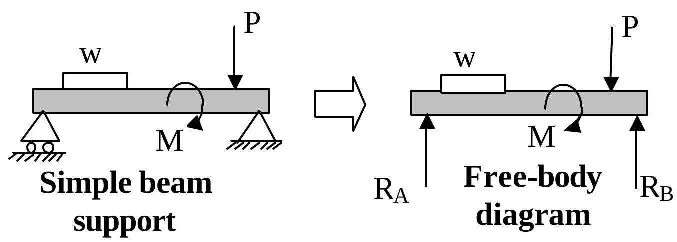
Transversely loaded members are often called beams. Beams can be supported as a cantilever beam or as a simple supported beam, as shown below.

Cantilever type support generates a vertical reaction force **R**, and a reaction moment $\mathbf{M_R}$ to support the beam. **Simply support** generates only vertical reaction forces, $\mathbf{R_A}$ **&** $\mathbf{R_B}$



The magnitudes and locations of **applied loads and moments** are known.

Types of loads:
P = Concentrated applied load
w= Distributed applied load
M = Applied moment



For these two types of supports, the unknown reaction force and reaction moment at the support can be readily determined by using the static equilibrium conditions.

6. SHEAR FORCE & BENDING MOMENT DIAGRAMS

Due to transverse loading, **shear forces and bending moments** are generated internally within beam.

The internal **shear force (V)** at any section of the beam:

V = sum of all external forces (including the reaction force at support) either to the left or right from the section. (Both will have same magnitude but opposite direction as the beam is in static equilibrium)

Sign convention of V = positive, if the force is upward to the left of the section.

To find the maximum V, often Vs along the length of the beam are determined and a diagram of variation of V is drawn, which is known as **shear force diagram**.

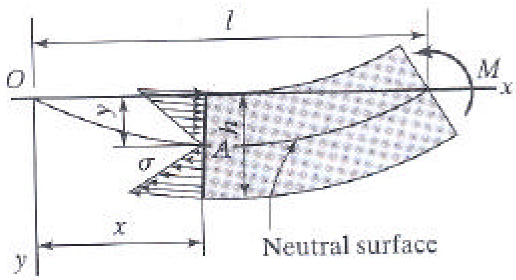
The internal **Bending Moment (M)** at any section of the beam:

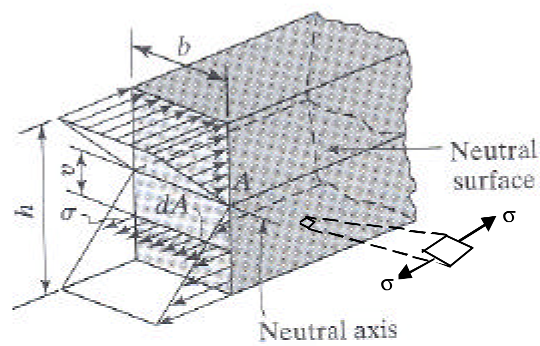
M = sum of moments about that section of all external forces (including the reaction force at support) either to the left or to the right from the section. (Both will have same magnitude but opposite direction as the beam is in static equilibrium)

Sign convention of M = positive, if the moment of a force causes compression in the upper layer of the beam.

To find the maximum M, often Ms along the length of the beam are determined and a diagram of variation of M is drawn, which is known as **bending moment diagram**.

7. BENDING STRESS (s)





Bending stress due to bending moment M will be zero at the neutral axis (NA). Bending stress is tensile in one side of the NA, and compressive on the other side of NA.

At any point in the loaded beam, bending stress (σ) can be calculated from the following formula:

$$s = \frac{M}{I} v$$

Where,
M = Internal bending moment at the point stress is being calculated,
I = Moment of Inertia of the beam cross section about the neutral axis (NA),
v = distance of the point from NA where the stress is determined.

Max bending stress will occur at the outermost layer of the beam (v=maximum), furthest away from the NA.

$$s_{\max} = \frac{M}{I} c$$

$$Also,\ s_{\max} = \frac{M}{Z}$$

Where,
c = distance of the furthest outer layer of the beam from the neutral axis (NA)
Z = I/c = known as section modulus

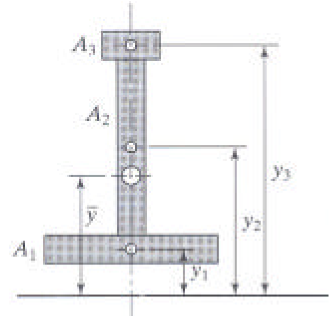
If the radius of curvature of the beam r, due to bending is known, bending stress (σ) can be found from the following alternative formula:

$$s = \frac{E}{r} v$$

Where,
E = Elastic Modulus
r = Radius of curvature due to bending at that section
v = distance of the point from the neutral axis (NA) where the stress is determined

8. NEUTRAL AXIS (NA)

NA passes through the **Center of gravity (CG)** of the beam cross section. For rectangular or circular cross-section of the beam, CG is at the geometric center of the section.

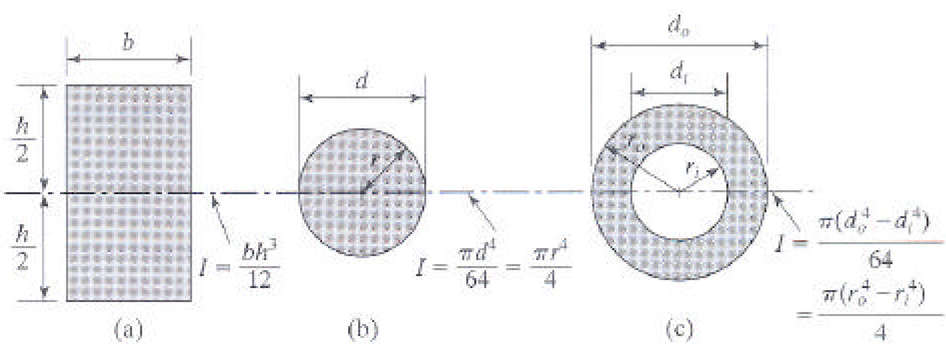


For a composite section, the **location of the CG** can be determined by the following formula,

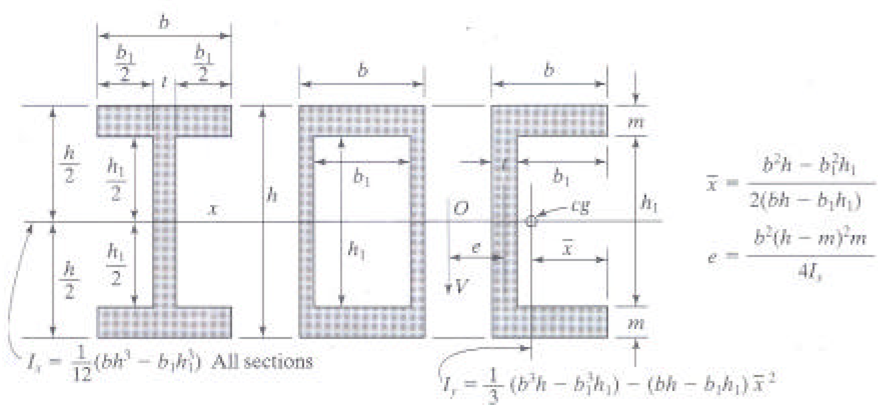
$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + \ldots}{A_1 + A_2 + A_3 + \ldots}$$

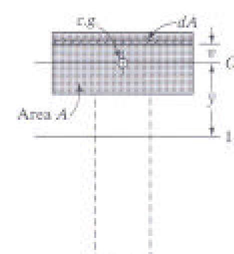
9. MOMENT OF INERTIA (I)

I, for rectangular and circular sections about their NA can be found using following formulae:



I for I-sections, Box sections and channel sections can be found using following formulae:





Transfer of axis for Moment of Inertia

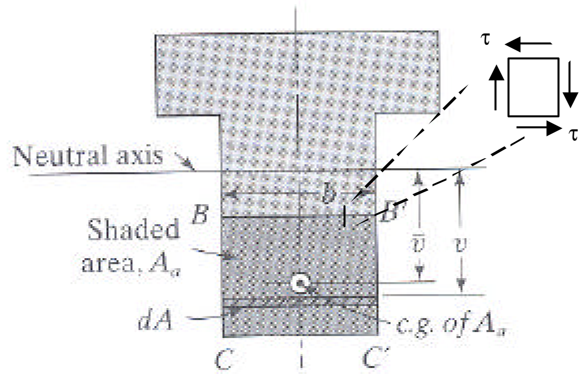
This formula is used to find MI of a T or other sections, whose NA or CG is not located at the geometric symmetrically.

$$I_1 = I_0 + A\bar{y}^2$$

10. TRANSVERSE SHEAR STRESS (t) IN BEAMS

(i) The transverse shear stress τ at a section BB’ is given by the following formula:

$$t = \frac{V}{Ib}\bar{v}A_a = \frac{VQ}{Ib}$$



Where,
V= Internal shear force,
I = MI about NA of the beam section
b= Width of the section
$\bar{v}$ = Distance of CG of the section BB’CC’ from NA
A_a= Area of the section BB’CC’
Q = $\bar{v}A_a$ = Area moment of the section BB’CC’

(ii) Max. transverse shear stress, τ_{max} always occurs at NA (because Q is max at NA)

(iii) When finding transverse shear stress (τ) for a composite section, the following formula can be used: $t = \frac{V}{Ib}\sum \bar{v}A_a = \frac{V}{Ib}\left(\bar{v}_1 A_{a1} + \bar{v}_2 A_{a2} ...\right)$

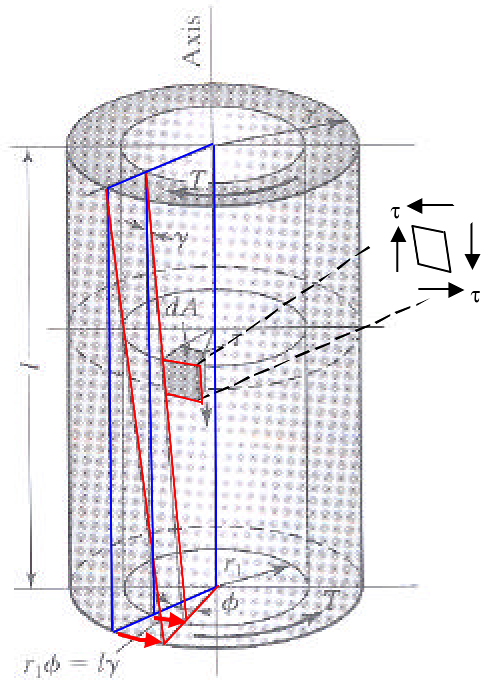
(iv) Max Transverse shear stresses (t_{max}):

Solid rectangular cross-section	$t_{max} = \frac{3}{2}\frac{V}{A}$	A = area of the cross-section = b.d
Solid circular cross-section	$t_{max} = \frac{4}{3}\frac{V}{A}$	A = area of the cross-section = $\frac{p}{4}d^2$
Circular cross section with thin wall*	$t_{max} = 2\frac{V}{A}$	A = area of the cross-section $= \frac{p}{4}\left(d_o^2 - d_i^2\right)$
I cross-section	$t_{max} = \frac{V}{A}$	A = t.d t= thickness of the web, and d= total depth of the I-beam

* A more exact analysis gives the values of 1.38 V/A and 1.23 V/A for the transverse shear stress at the center and ends, respectively, of the neutral axis.

11. TORSION OF CIRCULAR SECTION & SHEAR STRESS

When a torque T is applied to a **circular section**, the elastic deformation produces an angular twist φ, and shear stress τ, within the member. At any radial distance r_1, the shear stress τ, can be obtained from:



$$\tau = \frac{T}{J} r_1$$

Where,
J = polar moment of inertia of the circular section = 2I

The shear stress τ = 0 at the axis, as r_1=0, and the shear stress τ = τ_{max}, at r_1=r, the outermost layer.

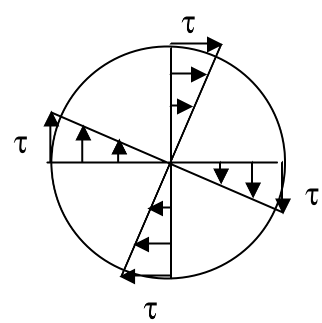
Thus, $\tau_{\max} = \dfrac{T}{J} r$

For, solid circular section: $J = \dfrac{\pi d^4}{32} = \dfrac{\pi r^4}{2}$

For, hollow circular section:

$$J = \frac{\pi (d_o^4 - d_i^4)}{32} = \frac{\pi (r_o^4 - r_i^4)}{2}$$

Putting the values of J, in the above equation:



For solid circular section:

$$\tau_{\max} = \frac{T}{J} r = \frac{T}{(\pi d^4/32)} \frac{d}{2} = \frac{16T}{\pi d^3}$$

For hollow circular section:

$$\tau_{\max} = \frac{T}{J} r = \frac{T}{(\pi (d_0^4 - d_i^4)/32)} \frac{d_o}{2} = \frac{16T}{\pi (d_0^4 - d_i^4)} d_o = \frac{16T}{\pi d_0^3 (1 - \lambda^4)}$$

$$where,\ \lambda = \frac{d_i}{d_o} = ratio\ of\ inner\ to\ outer\ diameter$$

The **shear stress** τ **and the angle of twist** φ is related by

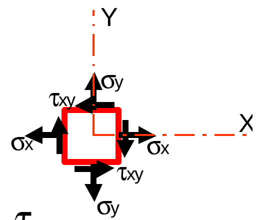
$\tau = \dfrac{\phi G r_1}{l}$, where G = shear modulus of elasticity, l = length of the shaft and r_1 = radius.

Relationship between power (kW/HP), torque (T) and rpm (n)

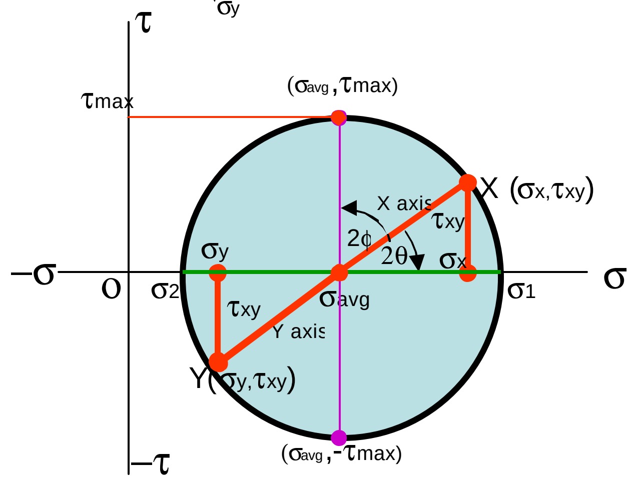
$$T = \frac{63{,}025 HP}{n}\ in - lb$$

$$T = \frac{9{,}550{,}000\ kW}{n} N - mm$$

12. EFFECT OF COMBINED STRESS IN 2D: MOHR CIRCLE



The two dimensional general state of stress at a point is shown, where, σ_x= normal stress in X direction, σ_y= normal stress in Y direction, and τ_{xy}= shear stress.



The corresponding Mohr circle can be drawn by plotting the X(σ_x,τ_{xy}), and Y (σ_y,τ_{xy}) points in the σ–τ plane. The circle drawn, using the XY line as diameter, is the Mohr circle.

The center of the circle $\sigma_{avg} = \dfrac{\sigma_x + \sigma_y}{2}$

Radius of the circle

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

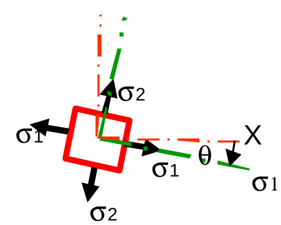
The angle, $2\theta = \tan^{-1}\left(\dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$, and the angle, $2\phi = 90 - 2\theta$

All Mohr circle angles are double the actual angles

Principal normal stresses will occur along the diameter $\sigma_1\sigma_2$ at an angle θ from the original X direction.

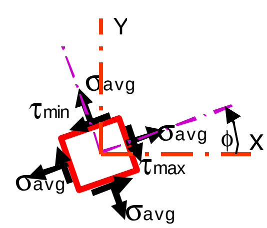
$$\sigma_1 = \sigma_{avg} + R = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \sigma_{avg} - R = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



Maximum shear stress will occur along the vertical diameter at an angle ϕ from the original X direction.

$$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



Stresses in any arbitrary direction u-v which is at an angle ϕ with respect to the x-y axes system, can be readily found if σ_{avg}, R & θ are known:

$$\sigma_u = \sigma_{avg} + R\cos[2(\theta+\phi)],\quad \sigma_v = \sigma_{avg} - R\cos[2(\theta+\phi)],\quad \text{and } \tau_{uv} = R\sin[2(\theta+\phi)]$$

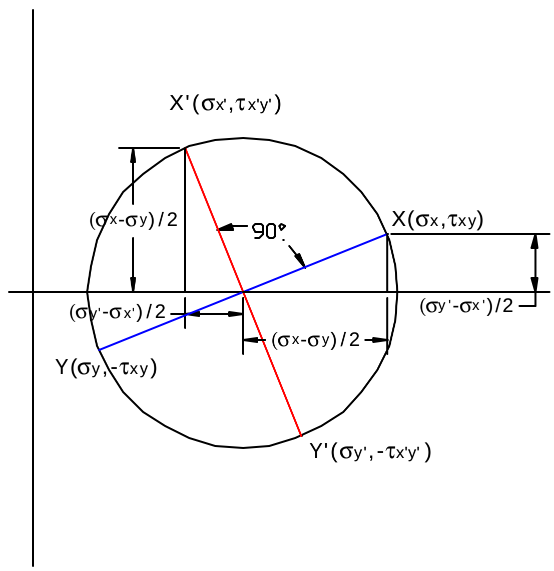
Stress relationship for a rotation of axis system by 45°

(a) When the XY coordinate system is rotated by **45° counter-clockwise** to X'Y' system

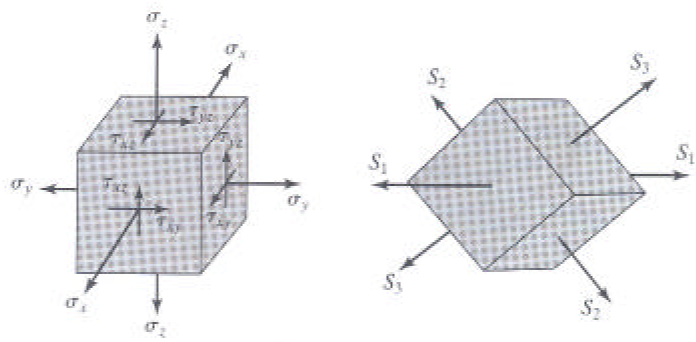
Then, $\tau_{x'y'} = \dfrac{\sigma_x - \sigma_y}{2}$, $\sigma_{x'} = \sigma_{avg} - \tau_{xy}$ and $\sigma_{y'} = \sigma_{avg} + \tau_{xy}$, where $\sigma_{avg} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{\sigma_{x'} + \sigma_{y'}}{2}$

(b) When the X'Y' coordinate system is rotated by **45° clockwise** to XY system

Then, $\tau_{xy} = \dfrac{\sigma_{y'} - \sigma_{x'}}{2}$, $\sigma_x = \sigma_{avg} + \tau_{x'y'}$ and $\sigma_y = \sigma_{avg} - \tau_{x'y'}$, where $\sigma_{avg} = \dfrac{\sigma_{x'} + \sigma_{y'}}{2} = \dfrac{\sigma_x + \sigma_y}{2}$



13. STRESS MANIPULATION IN 3-D



(a) General stress condition with shear stresses on each face.

(b) Principal stress condition with no shear stresses on any face.

For a general state of stress in 3D, shown in (a), normal and shear stresses may be present in 3 orthogonal directions. It can be shown that at a certain orientation, three principal normal stresses, orthogonal to each other, are equivalent to the stress condition at (a). S_1, S_2 and S_3 are the three principal normal stresses, which are three roots of the following cubic equation:

$$S^3 - a.S^2 + b.S - c = 0,$$

where, $a = \sigma_x + \sigma_y + \sigma_z$

$b = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2$

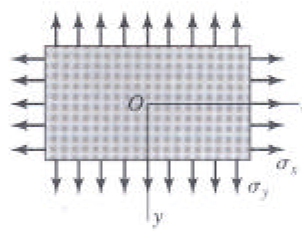
$c = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2$

The maximum shear stress $\tau_{max} = \text{Max of } \left(\frac{|S_1 - S_2|}{2}, \frac{|S_2 - S_3|}{2}, \frac{|S_3 - S_1|}{2} \right)$

14. STRAIN IN TWO DIMENSIONS

If the σ_x & σ_y are normal stresses, then normal strains ε_x & ε_y can be determined from:

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \mu\sigma_y), \textit{ and}$$
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \mu\sigma_x)$$



Where μ = Poisson's ratio, which is a material property

Solving the above two equations, we can find the normal stresses in two directions.

$$\sigma_x = \frac{E}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y), \textit{ and}$$
$$\sigma_y = \frac{E}{1-\mu^2}(\varepsilon_y + \mu\varepsilon_x)$$

15. THEORIES OF FAILURE

Uniaxial Tensile Stress Testing

During uniaxial stress testing of **ductile materials**, the first mechanical failure occurs by yielding, at the material's yield strength S_{yp}. The maximum stress that the material can withstand before breakage occurs at the ultimate tensile strength, S_u, and $S_u > S_{yp}$.
For ductile materials, S_{yp} and S_u values are same in tension and compression.

During uniaxial stress testing of **brittle materials**, the first mechanical failure occurs by fracture, at the material's ultimate tensile strength S_u. For brittle materials, S_{yp} is greater than S_u and thus S_{yp} is non-existent. Also, for brittle materials, fracture strength in compression S_{uc} is higher than fracture strength in tension S_{ut}.

General 3D state of stress

For these types of stresses, predicting failure is not as straight forward as in case of uniaxial stresses. The following theories of failure are developed to predict failure in such general state of stress. To apply these theories, **first the principal normal stresses S_1, S_2 and S_3 are computed**, and then the theories are applied, with a factor of safety N_{fs}. For 2D stresses, one of the principal normal stresses = 0.

A positive value of principal normal stress means the principal stress is tensile, and a negative value means that the principal stress is compressive.

Maximum Normal Stress theory Applicable for brittle materials	$\frac{S_{uc}}{N_{fs}} \le S_1 \le \frac{S_{ut}}{N_{fs}}$ $\frac{S_{uc}}{N_{fs}} \le S_2 \le \frac{S_{ut}}{N_{fs}}$ $\frac{S_{uc}}{N_{fs}} \le S_3 \le \frac{S_{ut}}{N_{fs}}$
Maximum Shear Stress theory Applicable for ductile materials	$\lvert S_1 - S_2 \rvert \le \frac{S_{yp}}{N_{fs}}$ $\lvert S_2 - S_3 \rvert \le \frac{S_{yp}}{N_{fs}}$ $\lvert S_3 - S_1 \rvert \le \frac{S_{yp}}{N_{fs}}$
Maximum Strain Energy: Applicable for ductile materials	$S_1^2 + S_2^2 + S_3^2 - 2m(S_1S_2 + S_2S_3 + S_3S_1) \le \left(\frac{S_{yp}}{N_{fs}}\right)^2$
Maximum Distortion Energy theory: Applicable for ductile materials (Also known as **von Mises-Hencky** theory)	$S_1^2 + S_2^2 + S_3^2 - S_1S_2 - S_2S_3 - S_3S_1 \le \left(\frac{S_{yp}}{N_{fs}}\right)^2$

16. STRESS CONCENTRATION

If there is a sudden change in geometry at a point, then the actual stress at that point is K times the calculated stress, where K>1. The value of K, the **stress concentration factor**, for combinations of geometry and loading type, can be obtained from the graphs given in pages 139-145 in the text book.

When the material is ductile, loads are not cyclic or are not applied suddenly or the application is not working in a low temperature condition, then the effect of stress concentration factors can be ignored, or K = 1, even if there is a sudden change in geometry.

For brittle materials all types of applications, and for ductile materials when, loads are applied suddenly, or for low temperature applications, the actual stress = K times the theoretically calculated stress.

For ductile materials, when the load is cyclic, a stress concentration factor K_f is used which is lesser than K.

K_f can be determined from the formula $q = \dfrac{K_f - 1}{K - 1}$.

Here, q is the **notch sensitivity factor or notch sensitivity index**, which depends on the material and its surface condition. The value of q can be obtained from handbooks and 0<q<1. If the value of q is unknown, then conservatively, it is assumed q=1, which gives K_f = K.

17. DESIGN FOR CYCLIC LOADING

For pure cyclic stress (S_r), that varies cyclically from 0 to S_r to 0 to $-S_r$,

$S_r K_f \le \dfrac{S_e}{N_{fs}}$, where S_e = endurance limit of the material, & N_{fs} = Factor of safety.

If the stress changes cyclically between S_{max} and S_{min}, then the equivalent steady stress (S_{avg}) and equivalent cyclic stress (S_r) can be given by $S_{avg} = \dfrac{S_{\max} + S_{\min}}{2}$ and $S_r = \dfrac{S_{\max} - S_{\min}}{2}$**. In such situations the design equations are as followings:**

Soderberg's equation: $S_{avg} + S_r K_f \left(\dfrac{S_{yp}}{S_e} \right) \le \dfrac{S_{yp}}{N_{fs}}$

Goodman's equation: $S_{avg} + S_r K_f \left(\dfrac{S_u}{S_e} \right) \le \dfrac{S_u}{N_{fs}}$

Modified Goodman's equation: $S_{avg} + S_r K_f \left(\dfrac{S_u}{S_e} \right) \le \dfrac{S_u}{N_{fs}}$ **&** $S_{avg} + S_r K_f \le \dfrac{S_{yp}}{N_{fs}}$

18. DESIGN FOR FINITE LIFE

(i) For a **completely reversing cyclic stress S_r**, the S-N curve is represented by,

$S_r = AN^B$, where N = number of stress reversals.

A & B are determined from:

$B = \dfrac{\log(S_e) - \log(0.9S_u)}{3}$, where S_e = endurance limit, and S_u = ultimate tensile strength.

$A = \dfrac{S_e}{10^{(6B)}}$

Once, A & B are known, for a given completely reversing stress σ_r **,** the number of stress reversals before failure can be found by: $N = \left(\dfrac{S_r}{A}\right)^{\frac{1}{B}}$

(ii) For a **combined reversing (S_r) and steady (S_{avg}) stress** situation, the equivalent completely reversing stress $S_R = \dfrac{K_f S_r S_{ult}}{S_{ult} - S_{avg}}$. For this type of loading, S_R should substitute S_r of the equation shown in (i)

(iii) **Miner's equation:** $\dfrac{n_1}{N_1} + \dfrac{n_2}{N_2} + \dfrac{n_3}{N_3} + ... = 1$**,** where n_1, n_2, n_3, … are actual number of reversals with S_{R1}, S_{R2}, S_{R3},….. equivalent completely reversing stress levels, and N_1, N_2, N_3, … are maximum number of reversals before failure with S_{R1}, S_{R2}, S_{R3},….. equivalent completely reversing stress levels.

19. SHEAR STRESS IN A SHAFT FOR COMBINED BENDING & TORSION

For a shaft carrying bending moment (M) and torque (T), the maximum shear stress (τ_{max}),:

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

For solid shaft: $\sigma = \dfrac{32M}{\pi d^3}$ *and* $\tau = \dfrac{16T}{\pi d^3}$ *thus* $\tau_{max} = \dfrac{16}{\pi d^3}\sqrt{M^2 + T^2}$

For hollow shaft:

$$\sigma = \frac{32M}{\pi d_o^{\,3}(1-\lambda^4)} \quad and \quad \tau = \frac{16T}{\pi d_o^{\,3}(1-\lambda^4)} \quad thus\ \tau_{max} = \frac{16}{\pi d_o^{\,3}(1-\lambda^4)}\sqrt{M^2+T^2}$$

$$where,\ \lambda = \frac{d_i}{d_o}$$

20. DESIGN OF SHAFT WITH CYCLIC LOAD

Based on maximum distortion energy theory, the design equation is:

$$\left(\sigma_{av} + \sigma_r K_{fb}\left(\frac{S_{yp}}{S_e}\right)\right)^2 + 3\left(\tau_{av} + \tau_r K_{ft}\left(\frac{S_{yp}}{S_e}\right)\right)^2 = \left(\frac{S_{yp}}{N_{fs}}\right)^2$$

Where,

σ_{av} = Steady normal stress = $\dfrac{\sigma_{max} + \sigma_{min}}{2}$

σ_r = Cyclic normal stress = $\dfrac{\sigma_{max} - \sigma_{min}}{2}$

K_{fb} = Fatigue stress concentration factor in bending

τ_{av} = Steady shear stress = $\dfrac{\tau_{max} + \tau_{min}}{2}$

τ_r = Cyclic shear stress = $\dfrac{\tau_{max} - \tau_{min}}{2}$

K_{ft} = Fatigue stress concentration factor in torsion

S_{yp} = Yield stress

S_e = Endurance limit

N_{fs} = Factor of Safety

21. SHAFT WITH BENDING LOADS IN TWO PLANES

(i) Resolve each bending load in vertical and horizontal direction.

(ii) Determine the bending moment diagram separately for horizontal and vertical loads

(iii) The resultant bending moment at any point on the beam = $M_R = \sqrt{M_v^2 + M_H^2}$

22. DESIGN OF KEYS, KEYWAYS & COUPLINGS

From the HP or kW rating and rpm n, torque T can be determined using the following formula:

$$T = \frac{63{,}025HP}{n}\ in - lb \quad or,\ T = \frac{9{,}550{,}000\ kW}{n}\ N - mm$$

Then, the tangential force F = T/r, where r = radius at which the tangential force is required.

(i) For designing **keys and keyways**, tangential force transmitted by the key is assumed to be acting at the outer diameter of the shaft, ie., $r = d_o/2$, and $F = 2T/d_o$

If, L = length of the key or the keyway
a = width of the key or the keyway, and
b = depth of the key

Then, **shear stress in the key** = $t = \frac{F}{A_s} = \frac{F}{a.L} \le \frac{t_{yp}}{N_{fs}}$,

It is assumed $t_{yp} = \frac{s_{yp}}{2}$, *thus*, $\frac{F}{a.L} \le \frac{s_{yp}}{2N_{fs}}$

Bearing stress in the key = $t = \frac{F}{A_b} = \frac{F}{\left(\frac{b}{2}\right)L} = \frac{2F}{bL} \le \frac{s_{yp}}{N_{fs}}\ or,\ \frac{F}{bL} \le \frac{s_{yp}}{2N_{fs}}$

For square Key, a=b, and hence key designed from either shearing or bearing stress, will result in same dimension, and square keys are equally strong from shearing and bearing.

(ii) **In rigid couplings**,
(a) Bolts may fail due to shearing or bearing:

Tangential force carried by each bolt = $F = \frac{2T}{nd_p}$, where, n = number of bolts, & d_p = pitch circle diameter of the bolts.

Shear stress in bolts = $t = \frac{F}{A_s} = \frac{4F}{p\ d_b^2} \le \frac{s_{yp}}{2N_{fs}}$

Bearing stress in bolts = $s = \frac{F}{A_b} = \frac{F}{td_b} \le \frac{s_{yp}}{N_{fs}}$,

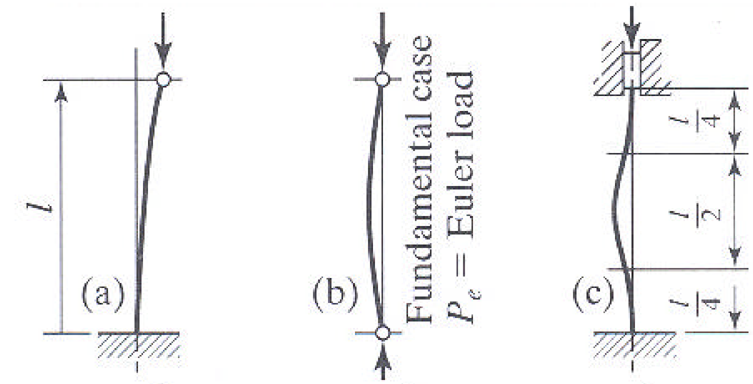
where, d_b= diameter of the bolt, and t = flange thickness of the coupling

(b) The coupling can shear from the hub/flange joint:

Tangential force $F = \frac{2T}{d_h}$, where d_h = hub diameter

Shear stress = $t = \frac{F}{A_h} = \frac{F}{p\ d_h} \le \frac{s_{yp}}{2N_{fs}}$

23. DESIGN OF SLENDER COMPRESSION MEMBERS OR COLUMNS



$$P_{cr} = \frac{\pi^2 EI}{4l^2} \qquad P_e = \frac{\pi^2 EI}{l^2} \qquad P_{cr} = \frac{4\pi^2 EI}{l^2}$$

Critical or buckling loads for centrally loaded columns.

For columns with an initial crookedness, the design load P can be determined from the following quadratic equation:

$$P^2 - \left[s_{yp} A + \left(1 + \frac{ac}{i^2} \right) P_{cr} \right] \frac{P}{N_{fs}} + \frac{s_{yp} A P_{cr}}{N_{fs}^2} = 0$$

Where, P = the design load
s_{yp} = yield strength
A = cross sectional area
a = initial crookedness
c = distance from the neutral axis to the edge of the cross section
$i = \sqrt{I / A}$ = radius of gyration
P_{cr} = Critical load for a centrally loaded column
N_{fs} = factor of safety.

Roots of quadratic equation $ax^2 + bx + c = 0; \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

24. DEFLECTION & SLOPE BY DOUBLE INTEGRATION

Express bending moment (M) as a function of the longitudinal distance, x of the shaft. i.e., M = f(x). If y represents downward deflection, then for a shaft with constant EI,

$$EI\frac{d^2y}{dx^2} = -M = -f(x) \text{ ....(1)}$$

Integrating both sides of (1):

$$EI\frac{dy}{dx} = -\int f(x) + C_1 \text{ ....(2)}$$

Here $\frac{dy}{dx}$ represents the slope of the shaft, and C_1 is the constant of integration. If the value of the slope is known for any value of x, then using those values, C_1 can be determined from the above equation.

Integrating both sides of (2):

$$EIy = -\int\int f(x) + C_1x + C_2 \text{ ....(3)}$$

Here y represents the downward deflection of the shaft, and C_2 is another constant of integration. If the value of the deflection y, is known for any value of x, then using those values, C_2 can be determined from the above equation. Usually y at the support = 0.

Once C_1 and C_2 are known, equation (2) and (3) are the slope and deflection equations for any valid value of x.

25. DEFLECTION & SLOPE BY STRAIN ENERGY METHOD

This method can take into account when diameter of the shaft is not uniform

Deflection $y = \frac{1}{E}\int\frac{M_pM_f dx}{I}$

and, slope $q = \frac{1}{E}\int\frac{M_pM_m dx}{I}$

Where M_p = Bending moment due to applied loads

M_f = Bending moment due to a fictitious unit load applied at the point where deflection is needed.

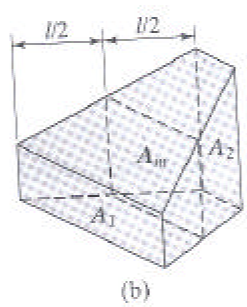
M_m = Bending moment due to a fictitious unit moment applied at the point where the slope is needed.

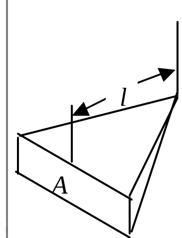
The above integrations can be calculated graphically. The integration is the volume enclosed by the $\frac{M_p}{I}$ and M_f diagrams for deflection, or the volume enclosed by the $\frac{M_p}{I}$ and M_m diagrams for slope for the entire length of the shaft.

For a prismoidal solid shown,

Volume = $\frac{l}{6}(A_1 + 4A_m + A_2)$

For a pyramid Volume = $\frac{l}{3}A$





26. SHAFT ON THREE SUPPORTS R1, R2 & R3

(i) Remove R2 and find the downward deflection, y of the shaft at R2, due to the downward applied loads.
(ii) Now, remove the applied loads and write an equation relating the upward force (R2) and upward deflection (y1) at the support point in the form R2=K.y1
(iii)Now three types of situations can arise:

(a) If all three supports are in the same level, then the magnitude of the reaction force
R2 = K.y

(b) If R2 is offset by δ from R1 & R3, then R2 = K(y-δ)

(c) If R2 is an elastic support with a spring constant K1 with no offset at no load, then:
(y-R2K1)K = R2 or, R2 = yK/(1+K.K1)

(iv) When the reaction force R2 is known, then the problem becomes statically determinate, and the R1 & R3 can be found by the application of static equilibrium conditions.

27. CRITICAL SPEED OF A ROTATING SHAFT

$$f = \sqrt{\frac{g(W_1y_1 + W_2y_2 + W_3y_3 + \text{...........})}{W_1y_1^2 + W_2y_2^2 + W_3y_3^2 + \text{......}}}\ cycles/\sec$$

$$n = 60 f\ r.p.m.$$

Where,
W_1, W_2, W_3 are the vertical loads on the shaft, and y_1, y_2, y_3 are deflections at of the loads, W_1, W_2, W_3 due to bending of the shaft.
g = acceleration due to gravity,
= 32*12 = 386 in/sec^2,
= 9806 mm/sec^2