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DIRECT NUMERICAL SIMULATION OF FERROFLUID DROPS IN A CYLINDRICAL MICROFLUIDIC UNDER THE INFLUENCE OF A NON-UNIFORM MAGNETIC FIELD

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ABSTRACT

Ferrofluids are magnetic liquids comprised of magnetic nanoparticles coated with surfactants and suspended in a carrier fluid. With biomedical and pharmaceutical applications in mind, we study the motion and deformation of axisymmetric ferrofluid droplets in the presence of non-uniform magnetic fields. Here we carry out direct numerical simulations of interfacial dynamics of ferrofluid droplet in a cylindrical microfluidic device driven by a magnetic field gradient. The governing equations are the Maxwell equations for a non-conducting flow, momentum equation and incompressibility. The numerical algorithm models the interface between a magnetized fluid and a non-magnetic fluid via a volume-of-fluid framework. A continuum-surface-force formulation is used to model the interfacial tension force as a body force, and the placement of the liquids is tracked via a volume fraction function.

INTRODUCTION

A ferrofluid is a liquid that becomes polarized in the presence of a magnetic field. It consists of magnetic nanoparticles in a colloidal solution. Interests in ferrofluids are motivated by biomedical and pharmaceutical applications. Recent developments in the synthesis and characterization of ferrofluids have enabled manufacturing of magnetite nano-particles coated with biocompatible polydimethylsiloxane (PDMS), with the advantages of a narrow size distribution, and colloidal stability [3]. In particular, this ferrofluid drop is synthesized for the potential treatment of retinal detachment [3]. The PDMS-nanoparticle complex is manufactured without the addition of a carrier fluid, which helps to minimize phase separation in applied fields.

To optimize the properties of ferrofluids for such applications, it is important to understand how droplets will behave in a fluid flow in response to an applied magnetic field. The motion and deformation of a ferrofluid droplet through a viscous medium can be quantified as its “magnetophoretic mobility”, a characteristic property of the droplet relating its velocity to the magnetic field and field gradient inducing the motion. The magnetophoretic mobility of a droplet is dependent upon both its magnetic properties and its size and shape. Here, we have developed a numerical model to study the deformation and the motion of ferrofluid droplets in a flow field in the presence of well-defined magnetic field and magnetic field gradient environments. The numerical model can therefore be used for the systematic understanding of relevant physics of magnetic drop formation in multiphase flows in microfluidic devices.

MATHEMATICS

The coupled motion of a ferrofluid drop suspended in a viscous fluid is governed by the Maxwell equations, the Navier-Stokes equations, and a constitutive relationship for the magnetic induction $\mathbf{B}$, magnetic field $\mathbf{H}$, and magnetization $\mathbf{M}$. The magnetostatic Maxwell equations for a non-conducting ferrofluid are: $\nabla\cdot\mathbf{B}=0$ and $\nabla\times\mathbf{H}=0$, where

$$\mathbf{B}(\mathbf{x},t)=\begin{cases}\mu_d\mathbf{H} & \text{in the ferrofluid drop}\\ \mu_m\mathbf{H} & \text{in the matrix,}\end{cases}$$

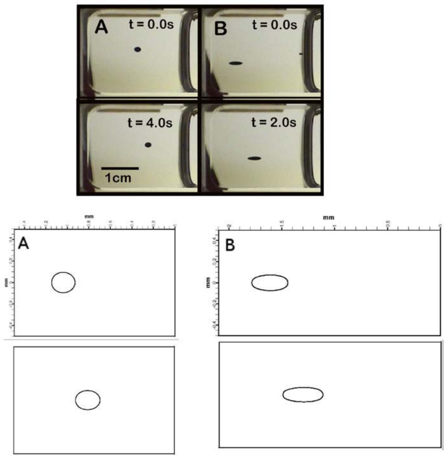

FIGURE 1. A PDMS-MAGNETITE FERROFLUID DROPLET IN A GLYCEROL MEDIUM, WITH A MAGNETIC FIELD GRADIENT OF 7.3 x 10^2 kA m^{-2}, (A) WITH A MEAN FIELD OF 1.8 kA m^{-1} AND (B) WITH A MEAN FIELD OF 18 kA m^{-1}. REPORTED FROM [4], EXPERIMENTAL PHOTOGRAPHIC IMAGES (TOP) AND NUMERICAL SIMULATIONS (BOTTOM).

where μ_d denotes the permeability of the drop, and μ_m is the permeability of the matrix fluid. For our application, the matrix fluid is glycerol which has a permeability very close to that for a vacuum, μ_o. A magnetic scalar potential ϕ is defined by $\mathbf{H} = \nabla\phi$, and satisfies $\nabla \cdot (\mu \nabla \phi) = 0$. We will assume a linear magnetization given by $\mathbf{M} = \chi \mathbf{H}$, where $\chi = (\mu/\mu_o - 1)$ is the magnetic susceptibility. The magnetic induction $\mathbf{B}$ is therefore $\mathbf{B} = \mu_o(\mathbf{H} + \mathbf{M}) = \mu_o(1+\chi)\mathbf{H}$. The fluid equation of motion is $\rho \frac{d\mathbf{u}}{dt} = \tau + \mathbf{F_s}$, where $\tau = -p\mathbf{I} + 2\eta\mathbf{D} + \tau_m$ and $\mathbf{F_s}$ denotes a body force. Here, p is pressure, $\mathbf{u}$ is velocity, $\mathbf{D} = \frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right)$ is the rate of deformation tensor where T denotes the transpose, η is viscosity, ρ is density, and τ_m is the magnetic stress tensor. The magnetic stress tensor of an incompressible, isothermal, linearly magnetizable medium is $\tau_m = -\frac{\mu}{2}H^2\mathbf{I} + \mu\mathbf{H}\mathbf{H}^T$, where $H = |\mathbf{H}|$. The density of the ferrofluid drop is matched with the surrounding liquid throughout. We refer the reader to and [2, 1] for details of the numerical approach.

NUMERICAL SIMULATIONS

The simulation results of the magnetic field-induced motion of PDMS ferrofluid droplets in a viscous medium are presented. The field gradient is varied enabling droplets to travel in the surrounding viscous medium. The boundary condition on the magnetic field is reconstructed by considering a magnetic field gradient of $\partial\mathbf{H}(0,z)/\partial z = 7.3$ x 10^2 kA m^{-2} with a mean field strength of $H = 1.8$ kA m^{-1} and 18 kA m^{-1}.

Two sets of computations are performed. A drop of 2 mm diameter is positioned at center of a computational domain. First a magnetic field gradient of 7.3 x 10^2 kA m^{-2} and a mean field of 1.8 kA m^{-1} is imposed. Second the same magnetic field gradient is used with a mean field of 18 kA m^{-1}. Figure 1 shows shapes of the droplet as it travels through the viscous medium. As shown, a higher velocity is computed for the larger magnetic field. Furthermore the droplet deforms from a sphere to form a prolate ellipsoidal shape. This illustrates how the magnetic field gradient in the domain also contributes to the deformation of the droplet. However, the shape of the droplet remains nearly spherical at the lower magnetic field strength. Figure 2 shows the streamlines and pressure distributions at t = 2 s. This figure provides further details on the motion of the droplet. The streamlines clearly demonstrate two different vortical flows in the viscous medium for droplet in different magnetic field gradients.

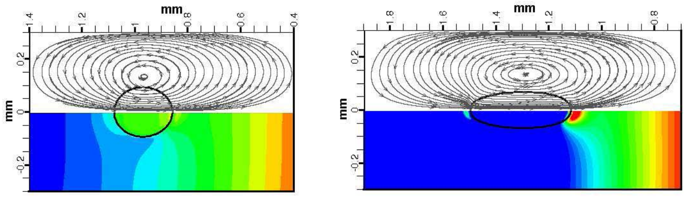

FIGURE 2. STREAMLINES (TOP) AND PRESSURE DISTRIBUTIONS (BOTTOM) AT t = 2 s. THE LEFT IMAGE IS WITH A MEAN FIELD OF 1.8 kA m^{-1} AND THE RIGHT IMAGE IS WITH A MEAN FIELD OF 18 kA m^{-1}.

SUMMARY

The effect of non-uniform magnetic fields on the deformation and motion of a bio-compatible hydrophobic ferrofluid drop suspended in a viscous medium is simulated numerically. Numerical results show qualitatively a similar behavior as experiments. We showed that the droplet undergoes deformation to form a prolate ellipsoid in the presence of an external magnetic field gradient. The numerical model can be used to optimize the magnetophoretic properties of ferrofluids.

ACKNOWLEDGMENTS

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