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EFFECTS OF VISCOSITY RATIO ON THE TRANSIENT AND STEADY DEFORMATION OF A NEWTONIAN DROP IN A VISCOUS AND VISCOELASTIC MATRIX UNDER SHEAR FLOW

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ABSTRACT

The deformation of a Newtonian drop suspended in a viscous and viscoelastic matrix under shear flow is investigated at viscosity ratio 1 and higher and polymeric to total viscosity ratio 0.5. Our direct numerical simulations reveal that for viscosity ratio 1 and capillary number 0.43, the steady drop deformation is less than the comparable Newtonian matrix for Weissenberg number 0.645, while as viscosity ratio increases past 3, the steady drop deformation is more than the comparable Newtonian matrix.

INTRODUCTION

For a viscoelastic matrix, the steady drop deformation is less than a comparable Newtonian matrix when the Weissenberg number is small, but increases with increasing Weissenberg number. Such non-monotonicity is not observed in experiments for small Weissenberg numbers. We carry out direct numerical simulations of Newtonian drop deformation in a viscous and viscoelastic matrix under shear flow. Here, we investigate the effects of the viscosity ratio on the transient and steady state deformation of the drop. We also present the transient and steady state drop inclination angle at viscosity ratio 1 and higher.

Transient 3D simulations are conducted with our in-house volume-of-fluid (VOF) code. The code uses a finite difference methodology on a regular Cartesian mesh. The placement of each fluid is determined by a volume fraction function for one of the liquids in each grid cell. The interface shape is reconstructed with piecewise quadratic reconstruction scheme, and is advected in a Lagrangian manner by the computed velocity field. We investigate the evolution of a Newtonian drop of initial radius R_0 and viscosity η_d, suspended in a viscoelastic matrix of total viscosity $\eta_m = \eta_s + \eta_p$ and retardation parameter $\beta = \eta_s/\eta_m$, where η_s and η_p denote the solvent and polymeric viscosity, respectively. The upper and lower boundaries of the domain are set into motion so that sufficiently far away from the drop, the flow is simple shear with shear rate $\dot{\gamma}$. The governing equations for each liquid are incompressibility $\nabla \cdot \mathbf{u} = 0$, and momentum transport

$$\rho(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}) = \nabla\cdot\big(-p\mathbf{I} + \mathbf{T} + \eta_s[\nabla\mathbf{u} + (\nabla\mathbf{u})^T]\big) + \mathbf{F}, \quad (1)$$

where $\mathbf{T}$ denotes the extra stress tensor and the surface tension force is computed as a body force $\mathbf{F}$ in the numerical formulation. The Oldroyd-B constitutive model for the matrix liquid is

$$\tau\Big(\frac{\partial \mathbf{T}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{T} - (\nabla\mathbf{u})\mathbf{T} - \mathbf{T}(\nabla\mathbf{u})^T\Big) + \mathbf{T} = \eta_p(\nabla\mathbf{u} + (\nabla\mathbf{u})^T), \quad (2)$$

where τ is the relaxation time. Drop deformation in the velocity-velocity gradient plane is defined by $D = (L-B)/(L+B)$, where L and B are the longest and shortest lengths from the center to the interface. The angle of inclination θ is defined to be the angle between the longest axis of the drop and the flow direction. The drop-to-matrix viscosity ratio is denoted $\lambda = \eta_d/\eta_m$. We define the capillary number $Ca = R_0\dot{\gamma}\eta_m/\Gamma$, where Γ is interfacial tension. A Reynolds number is defined by $Re = \rho\dot{\gamma}R_0^2/\eta_m$ and is small throughout. A Weissenberg number is $We = \dot{\gamma}\tau$. A dimensionless time is defined by $\hat{t} = t\dot{\gamma}$.

NUMERICAL RESULTS FOR DROP EVOLUTION

We use a computational domain of sides $L_x = 2L_y = 2L_z = 16R_0$ with x the flow, y the vorticity and z the velocity gradient

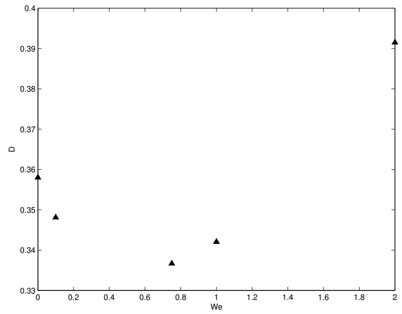

FIGURE 1. STEADY STATE DEFORMATION D AS A FUNCTION OF THE THE WEISSENBERG NUMBER FOR $\lambda = 1$ AND $Ca = 0.3$.

directions. We found that this domain size is sufficiently large to eliminate boundary effects. We use mesh size $\Delta x = \Delta y = \Delta z = R_0/12$ with the timestep of $\Delta\hat{t} = 0.0001$. We performed a spatial and temporal convergence test at $Re = 0.1, 0.05, 0.02$ and found that results are independent of the mesh size, timestep and Reynolds numbers.

Our simulations for viscosity ratio 1 and at $Ca = 0.3$ exhibit that the steady state deformation does not decreases monotonically with the increase of the Weissenberg number. Figure 1 plots the drop steady state deformation *vs.* the Weissenberg number, for viscosity ratio 1 and at $Ca = 0.3$. The curve has a minimum about $We = 0.8$ and then the drop deformation enhances with increasing We. This can be attributed to the viscoelasticity in matrix suppressing drop deformation for small We but increasing drop deformation at large We for $\lambda = 1$ [3, 2].

Next we look at the effect of varying viscosity ratio. Following our recent study [1], we present the results of drop deformation and rotation for $Ca = 0.43$, $We = 0.645$, when λ is varied form 1 to 8. We have observed in our numerical predictions that for $\lambda \geq 3$, the drop steady state deformation is enhanced with the matrix viscoelasticity, while for $\lambda < 3$, the drop steady state deformation is less when compared with the drop in a Newtonian matrix. Figure 2(A) shows transient drop deformations. Figure 2(B) shows the evolution of the drop inclination angle. Figure 2 demonstrates that when the matrix is viscoelastic, drop deforms less than a comparable Newtonian matrix at $\lambda = 1$ while the deformation becomes roughly equal at $\lambda = 4$ in both systems and increases in the viscoelastic matrix at $\lambda = 8$ when compared with a Newtonian matrix. However, the drop inclination angle is always more in a viscoelastic matrix than a comparable Newtonian matrix, suggesting the transient drop orientation angle evolves on a different time scale. We also note that the drop rotation is promoted when the viscosity ratio increases past 1, and this contributes to overshoots in transient deformation.

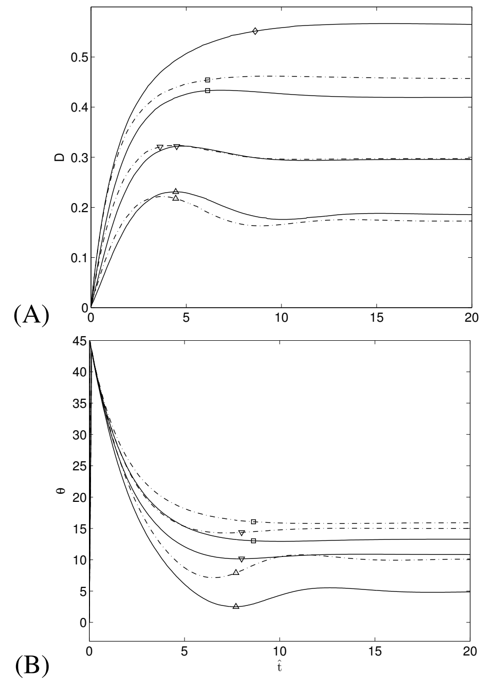

FIGURE 2. (A) DROP DEFORMATION D AND (B) DROP INCLINATION ANGLE θ *VS.* DIMENSIONLESS TIME, FOR λ = 1 (◇), 2 (□), 4 (▽), 8 (△) AT FIXED $Ca = 0.43$, $We = 0.645$(—) and $We = 0$(-.-). AT $\lambda = 1$ and $We = 0$, DROP IN A VISCOUS MATRIX DOES NOT REACH A STEADY STATE.

SUMMARY

We perform direct numerical simulation of a Newtonian drop in a viscous and viscoelastic matrix under shear flow. At viscosity ratio less than 3, the steady state deformation of a drop suspended in a viscous matrix is more than of when the drop is in a viscoelastic matrix. With increasing viscosity ratio with other parameters fixed, the viscoelastic effects tend to enhance the drop deformation. Further investigation of the viscoelastic and viscous stresses is required in order to explain this behavior in terms of the competition between viscoelastic and viscous forces at the interface.

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