

## Homework Assignment #10

**Problem 1.** Consider a semi-circular membrane of radius  $R$  satisfying

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u,$$

with zero displacement on the entire boundary and initially  $u(r, \theta, 0) = f(r, \theta)$  and  $\frac{\partial u}{\partial t}(r, \theta, 0) = g(r, \theta)$ .

- (i) Determine the solution.
- (ii) Determine the natural frequencies of oscillation.
- (iii) Find the solution if  $u(r, \theta, 0) = 0$  and  $\frac{\partial u}{\partial t}(r, \theta, 0) = \sin 5\theta$ .

**Problem 2.** Solve the heat equation

$$\frac{\partial u}{\partial t} = \kappa \nabla^2 u,$$

inside a circle of radius 1 with zero temperature around the entire boundary and initially  $u(r, \theta, 0) = f(r)$ . Analyze  $\lim_{t \rightarrow \infty} u$  as  $t \rightarrow \infty$ . Obtain the first term approximation for  $r \ll 1$  if initially  $u(r, \theta, 0) = 100(1-r)$  and graph the solution at different times on the same plot.