

Homework Assignment #3

Problem 1) 50 points

Consider a one dimensional rod of radius a and length L with heat loss through the lateral surface area. Assume a 0° surrounding temperature and a positive (and constant) convection heat transfer coefficient h . Consider both ends being perfectly insulated and the initial temperature distribution is $f(x)$.

- i) formulate the heat conduction within the rod.
- ii) find the equilibrium solution.
- iii) solve the time-dependent problem.
- iv) find $\lim u(x,t)$ as $t \rightarrow \infty$ and compare your result with the solution in part ii.
- v) find an approximate transient solution and justify your approximation.
- vi) use Matlab to plot the temperature at $x = 0.25$, $x = 0.5$, and $x = 0.75$ as a function of time, considering $c\rho = 1$, $K_0 = 1$, $L = 1$, $a = 0.1$, $h = 0.025$, and $f(x) = \begin{cases} 0 & x < 1/2 \\ 1 & x > 1/2 \end{cases}$.

Problem 2) 50 points

Suppose that you have a square plate of size 1 for which the temperature on two horizontal sides is kept at uniform temperature of 100° and on other two sides are kept at 0° . Find the solution and use Matlab to graph the temperature distribution on the plate. What is the temperature in the middle of the square?