Homework Assignment #4

Problem 1) 50 points

i) Write the Fourier Cosine and Sine series of $f(x) = \frac{k}{L}x \qquad 0 < x < L$.

ii) For $L, k = \pi$, plot the functions to which the series converge as well as the Fourier series for $n = 1,3,5,10$.

iii) State the theorem for the convergence of each series.

iiii) Show that the Fourier Cosine series can be differentiated term-by-term.

Problem 2) 50 points

i) Show that the forced vibrations of an elastic string are described by $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + \frac{P(x,t)}{\rho(x)}$, $0 < x < L,\ t > 0$, where $P(x)$ is force per unit length.

ii) Show that c^2 has the dimension of velocity squared.

iii) Consider both ends being fixed for $t > 0$ and $P(x) = -\rho(x)g$, where g is the gravitational acceleration. State the problem and find the equilibrium position of the string.