Homework Assignment #5

Problem 1) 50 points

The propagation of acoustic pressure p in a pipe of length L is governed by 1-D wave equation:

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

where c is the speed of sound and constant. Consider a closed left end and an open right end, i.e. $p(x = 0, t) = 0$ and $\frac{\partial p}{\partial x}(x = L, t) = 0$, respectively,

i) find the eigenvalues and the corresponding eigenfunctions
ii) find the normal modes
iii) write the frequencies of the normal modes
iv) using Matlab, plot the first, second, and third normal modes for various values of t

Problem 2) 50 points

Consider a case in problem 1 where initially $p = f(x)$ and $\frac{\partial p}{\partial t}$ is zero. Write the series solution for the propagation of acoustic pressure. Write the solution in the form of $p(x, t) = \frac{1}{2}[F(x - ct) + F(x + ct)]$ (hint: $\sin(a)\cos(b) = \frac{1}{2}[\sin(a + b) + \sin(a - b)]$). Show that $F(x)$ is the odd periodic extension of $f(x)$ with period of $2L$. Find the solution corresponding to $f(x) = \begin{cases} \frac{2k}{L}x, & 0 < x < \frac{L}{2} \\ k, & \frac{L}{2} < x < L \end{cases}$. Sketch $\frac{1}{2}F(x - ct)$, $\frac{1}{2}F(x + ct)$, and $\frac{1}{2}[F(x - ct) + F(x + ct)]$ for $t = 0, \frac{L}{5c}, \frac{2L}{5c}, \frac{L}{2c}, \frac{3L}{5c}, \frac{4L}{5c}, \frac{L}{c}$.