Homework Assignment #9

Problem 1. A thin square plate of size a is initially at 100° and allowed to cool with two of it sides maintained at 0° and two being insulated:

$$\frac{\partial u}{\partial t} = \kappa \nabla^2 u,$$

$$u(0, y, t) = u(a, y, t) = 0,$$

$$\frac{\partial u}{\partial x}(x, 0, t) = \frac{\partial u}{\partial x}(x, a, t) = 0$$

(i) Solve the initial-boundary value problem.
(ii) Find the smallest eigenvalue and the first term approximation.
(iii) Plot the isothermal contours at different times on separate graphs.
(vi) Compare your solution with the heat transfer in a rod of length a (i.e. one-dimensional problem) with ends maintained at 0° (and the lateral surface being insulated).
(v) Find the steady state temperature in the plate and compare your solution with the solution in part ii when $t \to \infty$.

Problem 2. The electric potential V in a domain follows the Laplace's equation. Find the potential in a box $0 \leq x \leq 10$, $0 \leq y \leq 20$, $0 \leq z \leq 5$ whose upper side is kept at 100 volts and with all the other sides grounded. What is the potential in the center of the box.