Matlab Project

Due: Nov. 18

Consider the following pseudocode

```
t = 0
dx = 1/K
for (i = 0 → K) do
  u_old[i] = f(i*dx)
end for
while (t < MAX TIME) do
  t = t + dt
  u_old[0] = g0(t)
  u_old[K] = g1(t)
  for (i = 1 → K − 1) do
    u_new[i] = u_old[i] + (k*dt/dx^2)*(u_old[i − 1] − 2*u_old[i] + u_old[i + 1])
  end for
  for (i = 1 → K − 1) do
    u_old[i] = u_new[i]
  end for
end while
```

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for a difference scheme used to approximate the solution of the one-dimensional IBVP for the heat equation

$$\frac{\partial U}{\partial t} = k\frac{\partial^2 U}{\partial x^2}, \quad x \in (0,1), \quad t > 0$$

$$U(x,0) = f(x), \quad x \in [0,1],$$

$$U(0,t) = g_0(t), \quad U(1,t) = g_1(t), \quad t \geq 0.$$

(I) Find the exact solution to the Initial-Boundary-Value problem for:

(a) $f(x) = 1$ (b) $f(x) = \sin 2\pi x$

and $g_0(t) = g_1(t) = 0$.

(II) Using the pseudocode, write a matlab code to approximate the solution of problems (I)a-b, for $k = 1/6$. Compare your results (by visualizing the solutions) with the exact solution at $t = 0.01, 0.1, 1, 10$ with $\Delta t = 0.01$ and taking $\Delta x = 1/10, 1/15, 1/20$. Find the largest value of Δt for $\Delta = 1/20$ that you can use to find a reasonable approximated solutions at $t = 10$.