Matlab Project

Due: Nov. 18

Consider the following pseudocode

\[
\begin{align*}
& t = 0 \\
& dx = 1/K \\
& \text{for } (i = 0 \rightarrow K) \text{ do} \\
& \quad u_{\text{old}}[i] = f(i*dx) \\
& \text{end for} \\
& \text{while } (t < \text{MAX TIME}) \text{ do} \\
& \quad t = t + dt \\
& \quad u_{\text{old}}[0] = g0(t) \\
& \quad u_{\text{old}}[K] = g1(t) \\
& \quad \text{for } (i = 1 \rightarrow K - 1) \text{ do} \\
& \quad \quad u_{\text{new}}[i] = u_{\text{old}}[i] + (k*dt/dx^2)(u_{\text{old}}[i-1] - 2*u_{\text{old}}[i] + u_{\text{old}}[i+1]) \\
& \quad \text{end for} \\
& \quad \text{for } (i = 1 \rightarrow K - 1) \text{ do} \\
& \quad \quad u_{\text{old}}[i] = u_{\text{new}}[i] \\
& \quad \text{end for} \\
& \text{end while}
\end{align*}
\]

for a difference scheme used to approximate the solution of the one-dimensional IBVP for the heat equation

\[
\frac{\partial U}{\partial t} = k \frac{\partial^2 U}{\partial x^2}, \quad x \in (0,1), \quad t > 0
\]

\[
U(x,0) = f(x), \quad x \in [0,1],
\]

\[
U(0,t) = g0(t), \quad U(1,t) = g1(t), \quad t \geq 0.
\]

(I) Find the exact solution to the Initial-Boundary-Value problem for:

\begin{enumerate}
\item[(a)] \( f(x) = 1 \)
\item[(b)] \( f(x) = \sin 2\pi x \)
\end{enumerate}

and \( g_0(t) = g_1(t) = 0 \).
(II) Using the pseudocode, write a matlab code to approximate the solution of problems (I)a-b, for $k = 1/6$. Compare your results (by visualizing the solutions) with the exact solution at $t = 0.01, 0.1, 1, 10$ with $\Delta t = 0.01$ and taking $\Delta x = 1/10, 1/15, 1/20$. Find the largest value of $\Delta t$ for $\Delta = 1/20$ that you can use to find a reasonable approximated solutions at $t = 10$. 
