1. Given the data points $\{(0,2),(1,5),(2,4)\}$,

(a) Find the piecewise linear interpolating function for the data.

(b) Find $P_2(x)$ that interpolates the data points.

(c) Find $P(x) = a + b\cos(\frac{\pi x}{2}) + c\sin(\frac{\pi x}{2})$ that interpolates the data points.

2. Determine the number of evenly spaced data points of the function $f(x) = \sqrt{x}$ for $1 \le x \le 2$, so that interpolation with $P_1(x)$ and $P_2(x)$ will yield an accuracy of 5×10^{-5}.

3. Is

$$s(x) = \begin{cases} x^3 - 3x^2 + 2x + 1, & 1 \le x \le 2 \\ -x^3 + 9x^2 - 22x + 17, & 2 \le x \le 3 \end{cases}$$

a cubic spline on $[1,3]$, and if so, is it a natural cubic spline function?

4. Let $f(x) = e^x$. Find a near-minimax polynomial $c_1(x)$ on $[0,1]$ and a bound on maximum error of near-minimax approximation.

5. Derive the triple recursion formula.

6. Consider $M_1(f) = (b-a)f(\frac{a+b}{2})$ to approximate $I(f) = \int_a^b f(x)\,dx$ on $[a,b]$. Determine the degree of precision of $M_1(f)$. Generalize this numerical integration to $M_n(f)$.

7. Give the rigorous and asymptotic error estimate for $I(f) - T_n(f)$ and $I(f) - S_n(f)$ for the numerical integration of

$$I(f) = \int_0^4 \frac{1}{1+x^2}\,dx.$$

Using the rigorous error bound, determine the number n for both T_n and S_n in order to have an accuracy of 5×10^{-10}.

8. Find a formula

$$\int_{-1}^{1} \frac{f(x)}{\sqrt{1-x^2}}\,dx \approx w_1 f(x_1),$$

where $f(x)$ has several continuous derivatives on $[0,1]$, that has a degree of precision equal to 1.