1. Use $P_3(x)$ with $x_j = x_0 + jh$ for $j = 0, 1, 2, 3$ to estimate $f'''(x_0)$. Also produce the associated error formula.

2. Find the inverse of

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix},$$

using Gaussian elimination with and without partial pivoting.

3. Consider

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 2 & 3 \end{bmatrix}.$$

(a) Show that $A\mathbf{x} = \mathbf{b}$ has a unique solution for any $\mathbf{b}$.
(b) When the Jacobi method is used, do you expect convergence? Explain.

4. Consider the spring system with resistance proportional to the square of velocity as

$$\frac{d^2y}{dt^2} + 0.1(\frac{dy}{dt})^2 + 0.6y = 0,$$
$$y(0) = 1,$$
$$y'(0) = 0.$$

Use Euler's method to find y at 0.1 with $h = 0.05$.

5. Problem 3 of Sec. 8.2.

6. Consider the problem

$$Y'(x) = -Y(x), \quad Y(0) = 1.$$

(a) Show that

$$e_{n+1} = (1-h)e_n + \frac{h^2}{2}e^{-c_n}.$$

(b) Using the result of part (a), show that

$$|e_n| \le \frac{x_n}{2}h.$$

(c) Compare this error bound to the true errors of approximating the solutions $y(0.1)$ and $y(0.2)$ with Euler's method with $h = 0.1$.

7. Determine whether the method

$$y_{n+1} = y_n + hf(\frac{x_n + x_{n+1}}{2}, \frac{y_n + y_{n+1}}{2}),$$

is absolutely stable.

8. Use the Runge-Kutta method of order 2 to find $y(0.1)$ and $y(0.2)$ with $h = 0.1$ and 0.05 for the problem

$$Y'(x) = -Y(x) + 2\cos(x), \quad Y(0) = 1.$$

Using the exact solution, show the true errors and the order of convergence of the results.

9. Find the real part of the region of stability for
(a) Adams-Bashforth method of order 2

$$y_{n+1} = y_n + \frac{h}{2}[3f(x_n, y_n) - f(x_{n-1}, y_{n-1})],$$

(b) Adams-Moulton method of order 2

$$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y_{n+1})].$$