A generalized Benford's law for JPEG coefficients and its applications in image forensics

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ABSTRACT

In this paper, a novel statistical model based on Benford's law for the probability distributions of the first digits of the block-DCT and quantized JPEG coefficients is presented. A parametric logarithmic law, i.e., the generalized Benford's law, is formulated. Furthermore, some potential applications of this model in image forensics are discussed in this paper, which include the detection of JPEG compression for images in bitmap format, the estimation of JPEG compression Q-factor for JPEG compressed bitmap image, and the detection of double compressed JPEG image. The results of our extensive experiments demonstrate the effectiveness of the proposed statistical model.

Keywords: Benford's law, first digit law, block-DCT coefficients, JPEG coefficients, probability distribution

1. INTRODUCTION

Digital image model has played an important role in digital image processing. A variety of statistical models have been proposed in literature for various image processing applications such as image filtering, image coding, image restoration, and image analysis to name a few. Moreover, many image processing applications are even impossible to pursue without appropriate statistical models. It is well known that the distribution of the Discrete Cosine Transform (DCT) coefficients can be modeled as Laplacian distribution [1] or Cauchy distribution [2]. To the best of our knowledge, however, the distribution of the most significant digits of the DCT coefficients has not been reported in the literature. In this paper, a novel statistical model based on Benford's law is proposed for the probability distributions of the first digits of the block-DCT coefficients and JPEG coefficients. Furthermore, some potential applications of this model are discussed in this paper.

Benford's law [3-5], also known as the first digit law or significant digit law, is an empirical law. It was first discovered by Newcomb [3] in 1881 and rediscovered by Benford [4] in 1938. Hill gave a statistical explanation of this law in [5]. It states that the probability distribution of the first digits, x ($x = 1, 2, ..., 9$), in a set of natural numbers is logarithmic. More specifically, if a data set satisfies Benford's law, its significant digits will have the following distribution:

$$p(x) = \log_{10}(1+\frac{1}{x}), x = 1, 2, ..., 9 \tag{1}$$

where $p(x)$ stands for probability of x.

Though it seems against the common sense at the first glance, the validity of Benford's law has been demonstrated and verified in various domains. While the naturally generated data obey the Benford's law well, the maliciously altered data and random guess data do not follow this law in general. This property has been widely used in fraud detection in accounting area [6, 7]. However, the applications of Benford's law in image processing field are only explored by some researchers in recent years. In [8], Jolion showed that the magnitude of the gradient of an image obeys this law and gives some possible applications in image processing such as entropy coding. In [9], Acebo and Sbert demonstrated how light intensities in natural images, under certain constraints, obey the Benford law closely. In this paper, we will show that the distribution of the most significant digits of the block-DCT coefficients follows Benford's law quite well and that of the

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quantized JPEG coefficients follow a Benford-like logarithmic law when the image has been JPEG compressed for only once. A parametric empirical model is proposed to formulate this law. Furthermore, we demonstrate that this distribution is very sensitive to the double JPEG compression, namely, the logarithmic law will be violated after double JPEG compression. This property can be favorably used in many image processing tasks including digital image forensics.

The rest of this paper is organized as follows. Section 2 illustrates that the first digit distribution of the block-DCT coefficients follows Benford's law closely. The distribution of the first digits of the quantized JPEG coefficients is presented in Section 3. Some potential applications of the proposed statistical model are proposed in Section 4 and conclusions are drawn in Section 5.

2. BLOCK-DCT COEFFICIENTS AND BENFORD'S LAW

JPEG compression is block-DCT based, and a popularly used image compression technology. A simplified diagram of the JPEG compression is illustrated in Fig. 1.

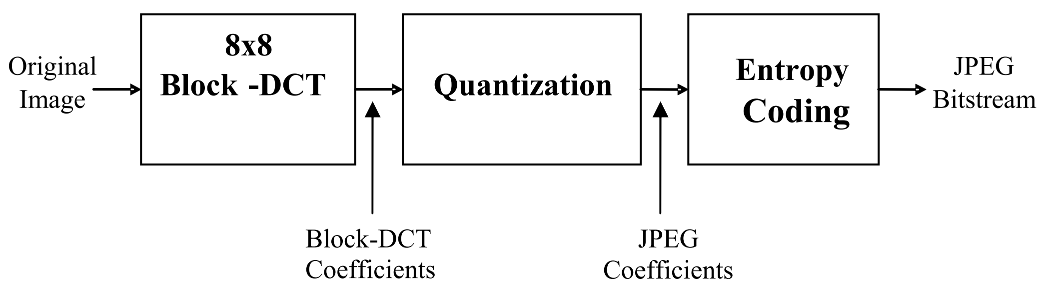


Fig. 1. Block diagram of JPEG compression.

As shown in Fig. 1, in the JPEG compression, the original image is first divided into non-overlapped 8x8 blocks. The two-dimensional DCT is then applied to each block. After that, the DCT coefficients are quantized using JPEG quantization table. In this paper, the term "block-DCT coefficients" is used to refer to the 8x8 block-DCT coefficients before the quantization and "JPEG coefficients" to refer to 8x8 block-DCT coefficients after the quantization with JPEG quantization table. Only the AC components of both the block-DCT coefficients and the JPEG coefficients are considered in this work if it is not stated explicitly otherwise. The probability distributions of both the block-DCT coefficients and JPEG coefficients are usually modeled as Laplacian distribution or Cauchy distribution. A variety of applications based on these models have been proposed. However, to our best knowledge, the distributions of the most significant digits of the block-DCT coefficients and JPEG coefficients have not been reported in the literature before. We will study the probability distribution of the first digits of the block-DCT coefficients in this section and that of the JPEG coefficients in the following section. For simplicity, we only consider 8-bit gray level images in this work. The same principle can, however, be easily extended to color images.

UCID (Uncompressed Image Database) [10], a publicly available uncompressed image database which consists of 1,338 uncompressed images (version 2) in tiff format is used in our experiments. The images in this database are quite diverse and are popularly used in image processing research community. Since the images in UCID are color images, only the luminance component is considered at this stage. To investigate the statistical characteristics of the block-DCT coefficients' first digit distribution, block-DCT is conducted for each image of UCID, and the frequencies of the first digits are then calculated. We discovered that the distribution of the first digits of the block-DCT coefficients follows the Benford's law quite well. Fig. 2 shows the experimental results.

In Fig. 2, the yellow (right) bars show the mean distribution of the first digits of the block-DCT coefficients of 1,338 images in the UCID database. The error bars (on the top of yellow bars) denote the standard deviations of the distributions for these images. The Benford's law is also illustrated in red (left) bars for comparison purpose. As can be seen, the probability distributions of the first digits of the block-DCT coefficients follow the standard Benford law very well. The quality of the fitting can be measured with the χ2 divergence [9]:

$$\chi^2 = \sum_{i=1}^{9} \frac{(\hat{p}_i - p_i)^2}{p_i} \tag{2}$$

where $\hat{p}_i$ is the actual first digit probability and p_i is the probability predicted by Benford's law as defined in Formula (1), namely, $p_i = \log_{10}(1 + \frac{1}{i})$. The average of χ2 divergences for the fitting of all the UCID images is only 0.0034, which indicates very good fitting results.

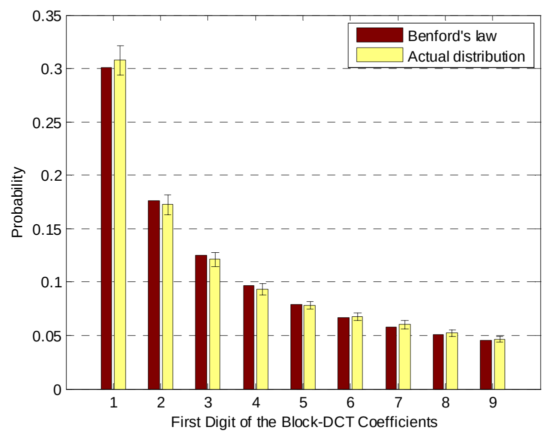
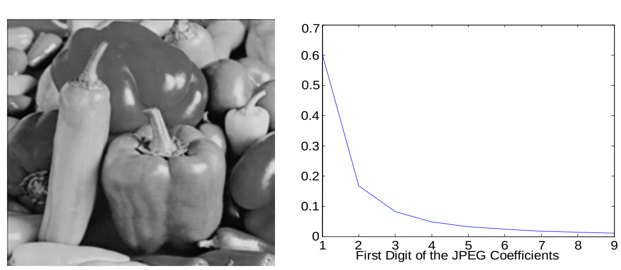


Fig. 2. The first digit distribution of the block-DCT coefficients for UCID database.

3. JPEG COEFFICIENTS AND GENERALIZED BENFORD'S LAW

In this section, the first digit distribution of the JPEG quantized block-DCT coefficients, namely, the JPEG coefficients is investigated.

Let *I* denote an image. Let *h(x)* be the probability mass function (pmf) or the normalized histogram of the JPEG coefficients of image *I*. The function *h(x)* has been typically modeled in generalized Laplacian distribution or Cauchy distribution, which have high peak in zero and long tails towards two sides horizontally. The distribution of the JPEG coefficients themselves is not the focus here. Instead, the distribution of the first digits of the JPEG coefficients is the subject under investigation in this paper.



(a) (b)

Fig. 3. (a) JPEG compressed Pepper image, (b) the distribution of the first digits of JPEG coefficients

As an example, Fig. 3 shows the Pepper image popularly used in image processing in (a), and the first digits distribution of its JPEG coefficients with Q-factor (Quality factor) 90 in (b). Compared with the histogram, the first digit distribution is much simpler because only 9 first digits (1, 2, …, 9) are involved. Also from Fig. 3 (b), we found that this distribution approximately follow a logarithmic law. This observation has been verified by our extensive experiments presented below.

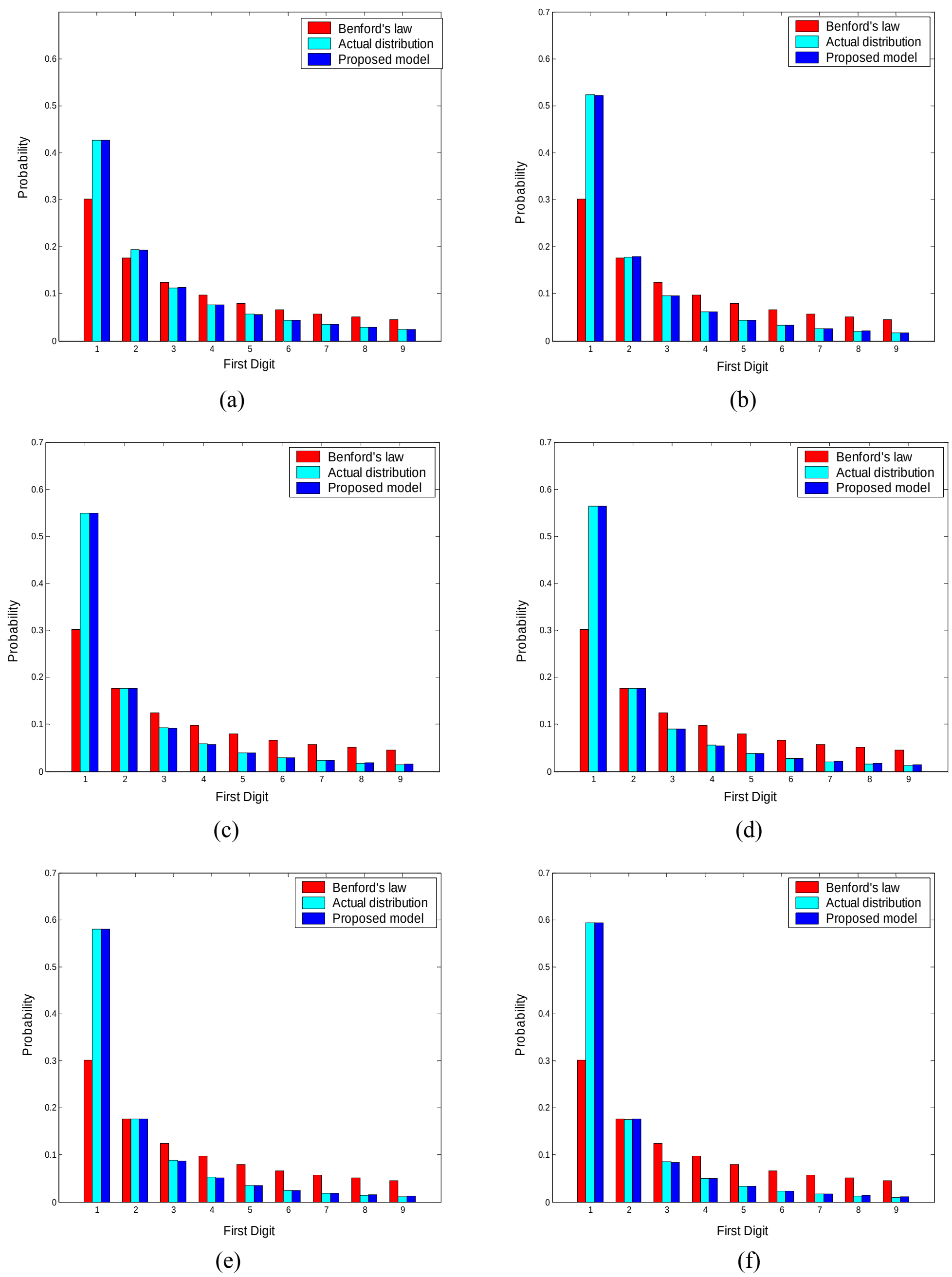


Fig. 4. Mean distributions of the first digits of JPEG coefficients under different JPEG Q-factors (QF): (a) QF=100; (b) QF=90; (c) QF=80; (d) QF=70; (e) QF=60; (f) QF= 50 (UCID database [10]).

Table 1 The fitting goodness of the proposed model for UCID database [8] (SSE: Sum of Squares due to Errors).

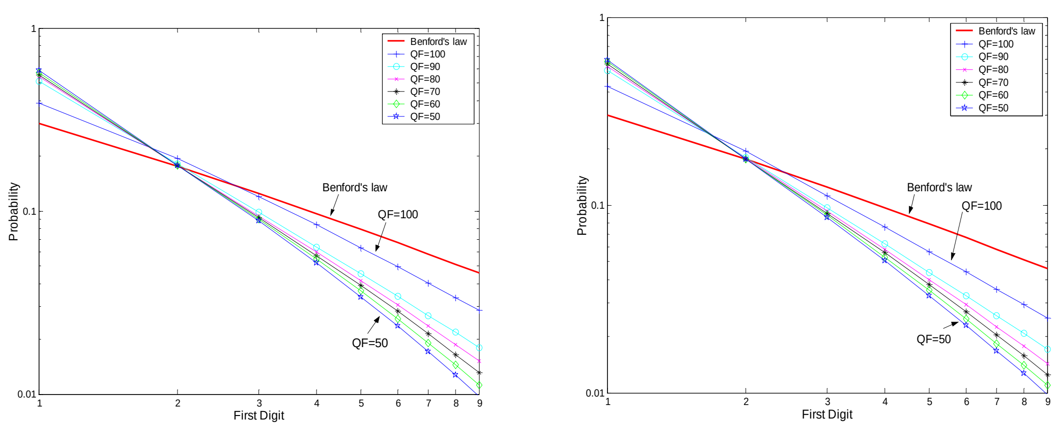
Q-factor	Model Parameters			Goodness-of-fit (SSE)
	N	q	s	
100	1.456	1.47	0.0372	7.104e-06
90	1.255	1.563	-0.3784	5.255e-07
80	1.324	1.653	-0.3739	3.06838e-06
70	1.412	1.732	-0.337	5.36171e-06
60	1.501	1.813	-0.3025	6.11167e-06
50	1.579	1.882	-0.2725	6.05446e-06

In our experiments, we still work on the publicly available uncompressed image database UCID [10] which consists of 1338 uncompressed images (version 2). Uncompressed image database used in this experiment guarantees that we know exactly the compression history of the images under investigation. To generate JPEG compressed images, we JPEG compress all the images in the above-mentioned database with a serial of different Q-factors: 100, 90, 80, 70, 60 and 50, respectively. In order to understand the statistical properties of the first digits of the JPEG coefficients in a JPEG compressed image, we averaged the distributions of the first digits of the JPEG coefficients obtained from 1,338 images of UCID database. Fig. 4 shows the mean distributions of the first digits of the JPEG coefficients under different JPEG compression Q-factors. For comparison purpose, we also plot the distribution of Benford's law as defined in Formula (1) in the figures. Comparing these distributions with the Benford's law, we observe that the distribution of the first digits of the JPEG coefficients does not follow the Benford's law in its rigorous form as shown in Formula (1). We also notice that these distributions still closely follow a logarithmic law. Therefore, we propose to model the distribution of the first digits of the JPEG coefficients by a parametric logarithmic function, called generalized Benford's law, as follows:

$$p(x) = N\log_{10}(1+\frac{1}{s+x^q}), x = 1,2,...,9 \tag{3}$$

where N is a normalization factor which makes $p(x)$ a probability distribution, s and q are model parameters to precisely describe the distributions for different images and different compression Q-factors. As we can see, when $s=0$ and $q=1$, Formula (3) reduces to Formula (1), which means that the distribution of Benford's law is just a special case of our proposed distribution model.

To illustrate the effectiveness of the proposed model, we provide the numerical fitting results in Table 1 for UCID database. The Matlab curve fitting tool box is used for data fitting in this paper. The Matlab toolbox returns a goodness-of-fit statistic called SSE (Sum of Squares due to Error). Other measures can also be used. The SSE's are only in the order of 10^{-6} . The fitting results of the proposed model are also illustrated in Fig. 4. As can be seen, the proposed model fits the actual mean distributions perfectly.



(a) (b)

Fig. 5. Mean distributions of the first digit of JPEG coefficients for different image databases (a) Harrison and (b) UCID.

In order to demonstrate the generality of the proposed model, we also perform the same experiments on another uncompressed dataset, Harrison, which consists of 198 images taken by our group members in Harrison, New Jersey with a Canon G2 camera and stored in RAW format. To illustrate these distributions in a more clear way, we plot them in log-log scale in Fig. 5 (all the rest figures are plotted in log-log scale for display purpose). That is, in Fig. 5, both the vertical axis and the horizontal axis are set in logarithmic scale. The distributions under different Q-factors (QF from 100 to 50) are displayed in same figure. We also plot the distribution of Benford's law for comparison. As we can see, although these two image datasets (Harrison and UCID) are quite different, the distributions of the first digits of the JPEG coefficients obviously follow the same law. We also notice that when Q-factor becomes small, the distribution of the first digits of the JPEG coefficients is almost a straight line in log-log space, which indicates that this distribution can even be approximated by a simple power law. This interesting phenomenon will be further investigated in our future work.

As shown in the above two sections, the first digit distribution of the block-DCT coefficients follows the standard Benford's law while that of the JPEG coefficients follows the proposed generalized Benford's law. In [11], we theoretically explained and experimentally demonstrated that it is the quantization process that causes this difference. It further demonstrates that it is the quantization that makes more severely monotonic decreasing of curves in Fig. 5 as Q-factor decreases.

4. APPLICATIONS IN IMAGE FORENSICS

We have shown in the above section that the distribution of the first digits of the JPEG coefficients of a JPEG compressed image follows a logarithmic law and its empirical model can be expressed as Formula (3). In this section, we demonstrate that this law will be violated if the image is double JPEG compressed by using different Q-factors. This favorable property can be used in many useful applications such as detection of the JPEG compression for bitmap image, estimation of JPEG Q-factor in JPEG compressed bitmap image, and detection of JPEG double-compression, etc.

4.1 Detection of the JPEG compression for bitmap image

To determine whether an image in bitmap format has been JPEG compressed previously or not is an important issue for some image processing applications [12]. It is also an important clue in image forensics. Given an image in bitmap format, there is no side information to tell the compression history. Even if it has been JPEG compressed before, there is no way to know and retrieve the compression quantization table through the format side information. We have to explore the characteristics of the image itself to identify the JPEG compression history of a given bitmap image. In [12], Fan and Queiroz proposed a JPEG compression detection scheme based on the detection of the blockiness artifacts introduced by JPEG compression. A maximum likelihood estimation method is proposed in their paper to estimate the JPEG quantization table after a JPEG image has been detected. Although their approach demonstrates some good results, its performance at very high compression quality (Q-factor >90) is rather limited and it fails when Q-factor is larger than 95. In this paper, we propose a novel JPEG compression detection algorithm based on the first digit distribution law we just discussed in the above section.

We work on the UCID database in this section. To prepare JPEG compressed image, we compress all these 1,338 uncompressed images into JPEG files by using different Q-factors 99, 95, 90, 80, 70, and 60, respectively. Then, all these compressed images are decompressed and stored in bitmap format again for experimental investigation. Our goal is to determine whether an image has ever been JPEG compressed or not. At this stage, we use Matlab *imwrite* function to perform the JPEG compressions. Standard JPEG quantization table is used in our experiments. For the JPEG image with non-standard quantization table, we believe that similar results can be expected. Our proposed JPEG compression detection scheme is based on the observation that the first digit distribution of the JPEG coefficients of single compressed image obeys the proposed generalized Benford's logarithmic law quite well while that of double compressed image does not. In this paper, we assume that the grid origin of 8x8 blocks is known. The block grid origin estimation algorithm is not a focus here.

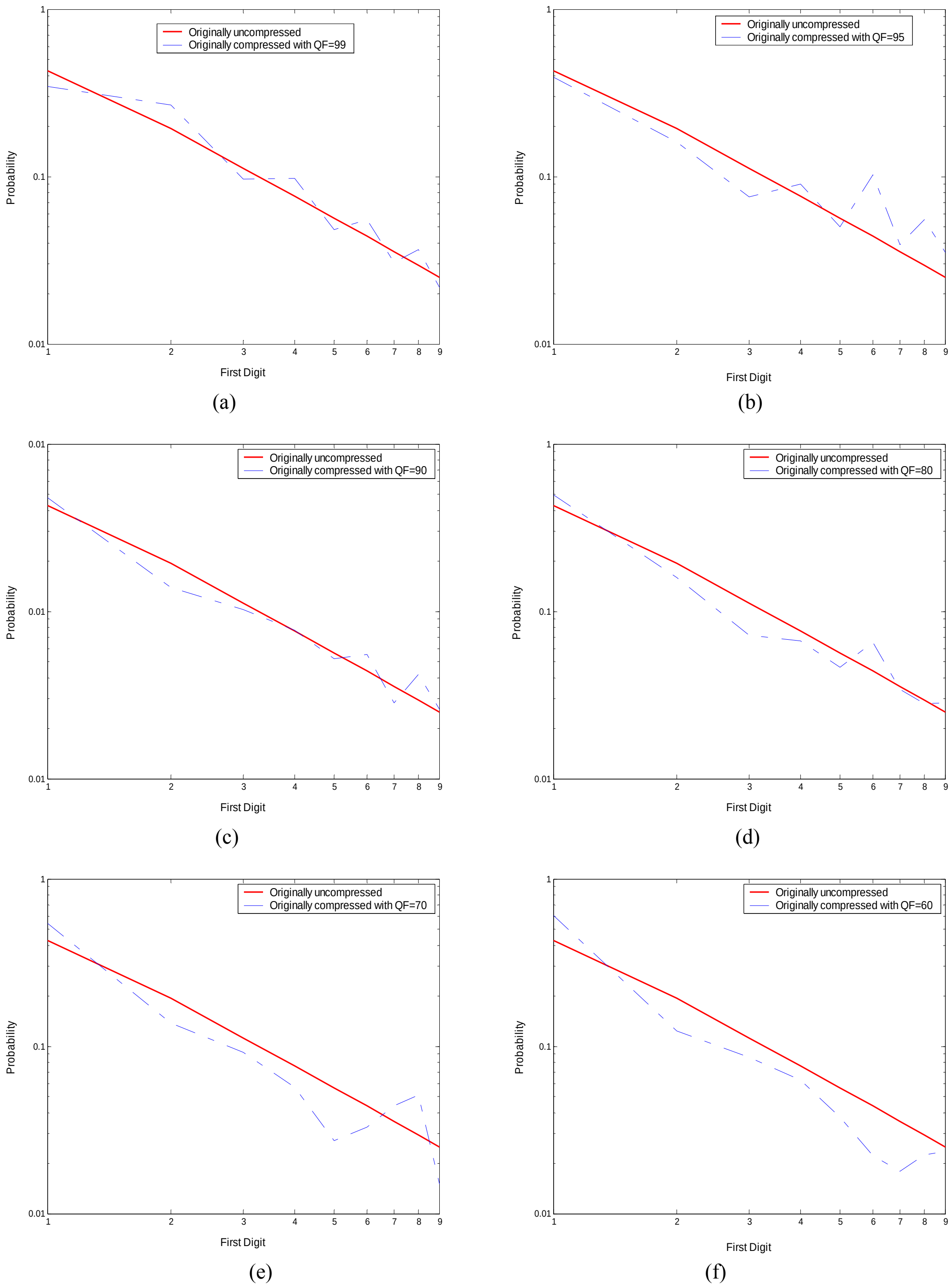


Fig. 6. Mean distribution of JPEG coefficients' first digits of the uncompressed bitmap images (red solids curve) and that of the JPEG compressed bitmap images with different Q-factors (QF) (blue dashed curve) (a) QF=99, (b) QF=95, (c) QF=90 ,(d) QF=80, (e)QF=70 and (f) QF=60 after re-compressed with JPEG Q-factor 100; refer to text for details (Log-Log scale is used in this Figure).

In the proposed scheme, for a given bitmap test image, we first compress it with JPEG Q-factor 100, which is the largest possible Q-factor in JPEG compression. In this way, if the given bitmap image has not been JPEG compressed previously, the resulting image is a single JPEG compressed image with Q-factor 100. The JPEG coefficients' first digit distribution is then obtained. This distribution should follow the proposed generalized Benford's (logarithmic) law perfectly as shown in last section. On the other hand, if the given bitmap image has ever been JPEG compressed previously, the resulting image is a double JPEG compressed image with the secondary Q-factor 100. In this case, the proposed logarithmic law will be obviously violated. Therefore we can discriminate the originally uncompressed image from the compressed one. As shown in Fig. 6, if the given image is an uncompressed image followed by JPEG compression with Q-factor 100 (i.e. single compressed with Q-factor 100), the logarithmic law is obeyed quite well (red curves). On the contrary, if the given image has been JPEG compressed with Q-factor as one of values among 99, 95, 90, 80, 70 and 60 followed by JPEG compression with Q-factor 100, obvious artifacts will show up in the first digit distribution of JPEG coefficients. For comparison reason, we plot the distribution of the first digit of the JPEG coefficients of originally uncompressed image after the process (JPEG compression with Q-factor 100) in each figure. To illustrate the statistical properties, all of curves in Fig. 6 actually show the mean distributions of the UCID database.

Since the artifacts are so obvious, many classification methods can be used to detect the JPEG compressed image. In addition to various learning algorithms, non-learning algorithms are also possible to work. For example, the goodness-of-fitting for the fitting of proposed model can be used to discriminate the compressed and uncompressed image. In our experiment, we simply use the logarithmic function of the first digit distributions as features and SVM (support vector machine) [13] is used as classifier. For the images in UCID database, 5/6 of them are used for training and the rest of 1/6 for testing. The detection results for different Q-factors are listed in Table 2.

Table 2. Performance of the proposed JPEG compression detection algorithm.

Q-factor	99	95	90	80	70	60
Detection Accuracy	100%	100%	100%	100%	100%	100%

As shown in Table 2, the proposed scheme can reliably detect JPEG compression with Q-factor as high as 99, which outperforms the method proposed in [12].

4.2 Estimation of Q-factor for JPEG compressed bitmap image

After a JPEG one-time compressed bitmap image has been identified, we further need to estimate the quantization table utilized in the JPEG compression because there is no such information in the bitmap file. In our scheme, we only estimate the overall Q-factor instead of individual elements of the quantization matrix at current stage. As stated previously, the standard JPEG quantization table is used in the experiments. For the JPEG image with non-standard quantization table, further investigation is needed. But we believe that similar results can be expected.

The estimation method is quite straightforward. The main idea is that when we re-compress the previously JPEG compressed image, the distribution of the first digits of the resulting image's JPEG coefficients will violate the proposed logarithmic law unless the re-compression Q-factor is equal to the original Q-factor. Fig. 7 shows an example to estimate the Q-factor of a given bitmap image whose actually used Q-factor is 80 (we randomly select an image from the UCID database and JPEG compress it with Q-factor 80). To estimate the Q-factor of this image, we first re-compress the image by a sequence of different Q-factors. The corresponding JPEG coefficients' first digit distributions are then calculated. The Q-factor associated with the JPEG re-compression having the least distribution artifacts compared with the generalized Benford's logarithm model is chosen as the Q-factor estimate of the given image. We also illustrated the numerical fitting results in Table 3. As can be seen, Q-factor 80 has the least fitting error. Q-factor 80 is then chosen as the estimation of the Q-factor of the test image. The estimation is correct and accurate in this example.

There is one thing that needs to be mentioned. Similar to the phenomenon in the histogram of the double-compressed JPEG image pointed out in [14, 15], when the re-compression quantization step size is exactly integer times of the original compression quantization step size, there is no artifacts shown in the first digit distribution of the double-compressed JPEG coefficients. Therefore, we must re-compress the given test image with various Q-factors, starting from the highest Q-factor 100 with gradually and monotonically decreasing Q-factors. Finally, the highest Q-factor associated with distortion below a threshold is selected as the estimate of the original Q-factor.

Table 3. Fitting results for different re-compression Q-factors.

Q-factor	Model Parameters			Goodness-of-fit (SSE)
	N	q	s	
85	0.1966	0.475	-0.9982	0.0082
83	0.4738	0.9793	-0.9262	0.00072
80	**1.227**	**1.571**	**-0.4212**	**9.75616e-06**
77	1.569	1.719	-0.144	0.00016
75	3.537	2.388	1.41	0.00033

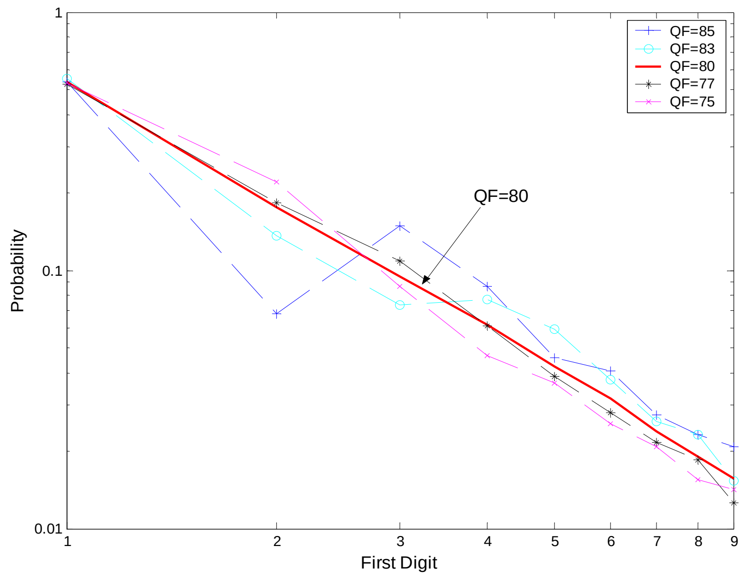
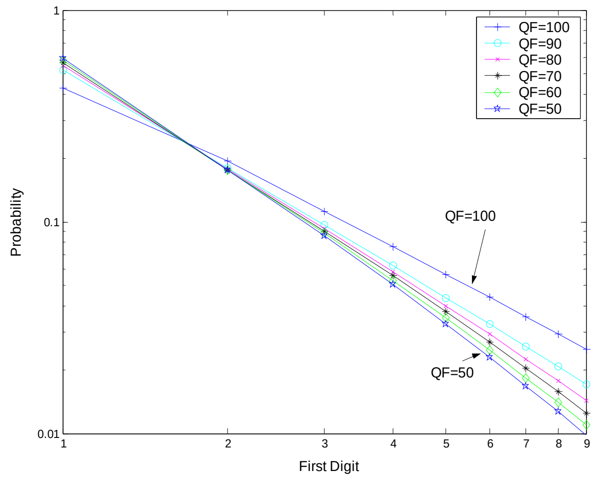


Fig. 7. Estimation of the JPEG compression Q-factor for bitmap image.

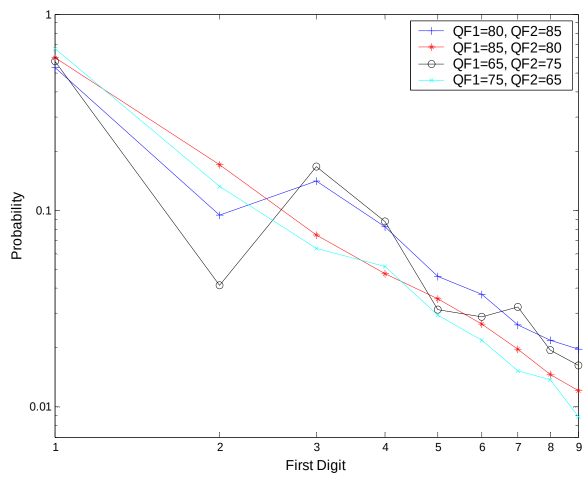
4.3 Detection of JPEG double-compression image

JPEG double-compression is an important issue in image steganalysis [14] and forgery detection [15]. Although there are detection methods proposed in literature [e.g., 14, 15], their methods are all based on the histogram artifacts introduced by the JPEG double-compression. As shown in Fig. 8, double-compression also causes severe violation of the first digit law proposed in this paper. Based on this fact, JPEG double-compression can be reliably detected by exploiting the artifacts in the distribution of the first digits of JPEG coefficients. Similar to the discussions in Section 4.1, either machine learning or non-learning algorithms can be used to discriminate the double-compressed JPEG image from the single compressed JPEG image.

More investigations on this topic and on estimation of the primary Q-factor in double compressed JPEG image are our future work.



(a)



(b)

Fig. 8. Some distributions of the first digits of JPEG Coefficients for (a) single-compressed and (b) double-compressed images (QF1: primary Q-factor; QF2: secondary Q-factor) (UCID database) (Log-Log scale).

5. CONCLUSIONS

We have presented a novel statistical model for the distribution of the first digits of the block-DCT and quantized JPEG coefficients. The main contributions of this paper are as follows.

It is the first time in literature that the distribution of the first digits of the block-DCT coefficients and JPEG coefficients has been explored. An effective parametric logarithmic distribution model, the generalized Benford's law, has been proposed. The extensive experiments have shown the effectiveness of the proposed model. Some image forensic applications of the proposed model have been discussed.

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