Introduction

- A significant portion of communication today (1998) is in analog.
- It is being replaced rapidly by digital communication.
- Within the next decade, most of the communication will become digital, with analog communication playing a minor role
- This chapter addresses various aspects of digital data transmission
Digital Communication Systems: Source

- Source, Multiplexer, Line Coder, Regenerative Repeater, Receiver (Detection), …

- Source: The input
  - A sequence of digits
  - We discuss mainly: The binary case (two symbols)
  - Later on: The M-ary case (M symbols)
    more general case

Multiplexer

- Usually, the capacity of a practical channel $\gg$ the data rate of individual sources
- To utilize this capacity effectively, we combine several sources through a digital multiplexer using the process of interleaving
- A channel is time-shared by several messages simultaneously
Line Coder (Transmission Coder)

• The output of a multiplexer is coded into electrical pulses or waveforms for the purpose of transmission over the channel.

• This process is called a line coding or transmission coding.

• Many possible ways of assigning waveforms (pulses) to the digital data.

• On-off:

  \[
  \begin{align*}
  1 & \iff a \text{ pulse } p(t) \\
  0 & \iff \text{ no pulse}
  \end{align*}
  \]

Line Coder …

• Polar:

  \[
  \begin{align*}
  1 & \iff a \text{ pulse } p(t) \\
  0 & \iff a \text{ pulse } -p(t)
  \end{align*}
  \]

• Bipolar (pseudoternary or alternate mark inversion (AMI))

  \[
  \begin{align*}
  1 & \iff \text{ a pulse } p(t) \text{ or } -p(t) \text{ depending on whether the previous } 1 \\
  0 & \iff \text{ no pulse }
  \end{align*}
  \]

  In short, pulses representing consecutive 1’s alternate in sign.

• All the above three have used half-width pulses for the sake of illustration. It is possible to select other width.

• Figure 7.1 (parts a, b and c).
Line Codes

- Full width pulses are often used in some applications.
  - i.e., the pulse amplitude is held to a constant value throughout the pulse interval (it does not have a chance to go to zero before the next pulse begins).
  - These schemes are called non return-to-zero (NRZ) in contrast to return-to-zero (RZ).
  - Figure 7.1 shows
    - An On-off NRZ signal
    - A polar NRZ signal in d and e parts of the figure

Figure 7.1 Some line codes. (a) On-off (RZ), (b) Polar (RZ), (c) Bipolar (RZ), (d) On-off (NRZ), (e) Polar (NRZ).
Regenerative Repeater

- Used at regularly spaced intervals along a digital transmission line to: 1) detect the incoming digital signal and 2) regenerate new clean pulses for further transmission along the line.
- This process periodically eliminates, and thereby combats the accumulation of noise and signal distortion along the transmission path.
- The periodic timing information (the clock signal at $R_b$ Hz) is required to sample the incoming signal at a repeater.
- $R_b$ (rate): pulses/sec

Regenerative Repeater…

- The clock signal can be extracted from the received signal.
  - e.g., the polar signal when rectified ⇒ a clock signal at $R_b$ Hz
- The on-off signal = A periodic signal at $R_b$ + a polar signal (Figure 7.2)
- When the periodic signal is applied to a resonant circuit tuned to $R_b$ Hz, the output, a sinusoid of $R_b$ Hz, can be used for timing.
On-off Signal Decomposition

Regenerative Repeater...

- The bipolar signal \(\text{rectified}\) an on-off signal \(\Rightarrow\) the clock signal can also be extracted.
- The timing signal (the output of the resonant circuit) is sensitive to the incoming pattern, sometimes.
  - E.g. in on-off, or bipolar \(\{0 \leftrightarrow \text{no pulse}\}\)
- If there are too many zeros in a sequence, will have problem.
  - no signal at the input of the resonant circuit for a while
  - sinusoids output of the resonant circuit starts decaying
- Polar scheme no such problem.
Transparent Line Code

• A line code in which the bit pattern does not affect the accuracy of the timing information is said to be a transparent line code.
  – The polar scheme is transparent.
  – The on-off and bipolar are not transparent (non-transparent).

Desired Properties of Line Codes

1. Transmission bandwidth: as small as possible
2. Power efficiency: for a specific bandwidth and detection error rate, transmitted power should be as small as possible.
3. Error detection and correction capability: as strong as possible
4. Favorable power spectral density: desirable to have zero PSD at $\omega = 0$ (dc), ac coupling is required. If at dc, $PSD \neq 0$, dc wanders in the pulse stream.
5. Adequate timing content: should be possible to extract the clock signal (timing information).
6. Transparency
PSD of Various Line Codes

• Consider a general PAM signal, as shown in Figure 7.3 (b)
  \[ R_b \quad T_b = \frac{1}{R_b} \quad t = kT_b \]
  – the \( k \)th pulse in the pulse train \( y(t) = a_k p(t) \)
  • \( p(t) \): the basic pulse
  • \( P(\tau) [P(f)]: \) Fourier spectrum of \( p(t) \)
  • \( a_k: \) arbitrary and random
  – The on-off, polar, bipolar line codes are all special cases of the general pulse train \( y(t), a_k = 0, +1, -1 \)

A Random PAM signal

Figure 7.3  A random PAM signal and its generation from a PAM impulse sequence.
PSD of Various Line Codes …

• How to determine the PSD?
  – Figure 7.3
  – An attractive general approach
    \( R_x(\tau), S_x(\omega) \) need to be studied once
    Then for different \( p(t) \), \( \Rightarrow \) different PSD
  – \( x(t) \): an impulse train
  – \( \hat{x}(t) \): a rectangular pulse train \((\varepsilon \cdot h_k = a_k)\)
    When \( \varepsilon \rightarrow 0, \hat{x}(t) \rightarrow x(t) \)
    (Figure 7.4)
PSD of Various Line Codes …

\[ R_0 = \overline{a_k^2} \quad \Rightarrow \quad \text{time average of} \quad a_k^2 \]

\[ R_1 = a_k.a_{k+1}, \ldots, R_n = a_k.a_{k+n} \]

\[ R_X(\tau) = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n \delta(\tau - nT_b) \]

\[ S_X(\omega) = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n e^{-j n \omega T_b} = \frac{1}{T_b} \left[ R_0 + 2 \sum_{n=1}^{\infty} \left[ R_n \cos(n \omega T_b) \right] \right] \]

\[ S_y(\omega) = |P(\omega)|^2 S_X(\omega) \]

Different line codes \(\Rightarrow\) different \(P(\omega)\) \(\Rightarrow\) different \(S_y(\omega)\)

---

Polar Signaling

\[ 1 \leftrightarrow p(t) \quad 0 \leftrightarrow -p(t) \]
\[ a_k = \pm 1, \text{equally likely}, a_k^2 = 1 \]

\[ R_0 = \overline{a_k^2} = 1 \]
\[ R_1 = a_k.a_{k+1} = 0 \quad \therefore a_k.a_{k+1} = 1 \quad \text{or} \quad -1 \]
\[ R_n = 0, n \geq 1 \quad \text{equally likely} \]
\[ \text{Similar reasoning} \]

\[ S_y(\omega) = |P(\omega)|^2 S_X(\omega) \]
\[ = \left| P(\omega) \right|^2 \frac{1}{T_b} \]
Polar Signaling…

• To be specific, assume $p(t)$ is a rectangular pulse of width $\frac{T_b}{2}$ (a half-width rectangular pulse)

$$p(t) = \text{rect} \left( \frac{t}{T_b} \right)$$

$$P(\omega) = \frac{T_b}{2} \text{Sinc} \left( \frac{wT_b}{4} \right)$$

$$S_y(\omega) = \frac{T_b}{4} \text{Sinc}^2 \left( \frac{wT_b}{4} \right)$$

**1st zero-crossing:**

$$\omega T_b = \pi \Rightarrow \omega = \frac{4\pi}{T_b}$$

$$\Rightarrow f = \frac{2}{T_b} = 2R_b$$

---

**PSD of A Polar Signal**

![Power spectral density of a polar signal](image)

Figure 7.5: Power spectral density of a polar signal.
Polar Signaling …

• Comment 1: The essential bandwidth of the signal (main lobe) = 2 R_b For a full-width pulse, \( \Rightarrow R_b \) “Polar Signaling is not bandwidth efficient.”

• Comment 2: “No error-detection or error-correction capability.”

• Comment 3: “Non-zero PSD at dc( ? = 0)” \( \Rightarrow \) This will rule out the use of ac coupling in transmission.

• Comment 4: “Most efficient scheme from the power requirement viewpoint” \( \Rightarrow \) for a given power, the detection-error probability for a polar scheme is the smallest possible.

• Comment 5: “Transparent”

Achieving a DC Null in PSD by Pulse Shaping

\[ \therefore P(\infty) = \int_{-\infty}^{\infty} p(t)e^{-j\omega t} \, dt \]

\[ \therefore P(0) = \int_{-\infty}^{\infty} p(t) \, dt \]

\[ \Rightarrow \text{If the area under } p(t) = 0, P(0) = 0. \]
Manchester
(Split-phase, twinned-binary) Signal

On-off Signaling

- \( 1 \leftrightarrow p(t) \)
- \( 0 \leftrightarrow \text{no pulse} \)

\[
R_0 = \frac{1}{2}
\]
\[
R_n = \frac{1}{4} \quad \text{for all} \quad n \geq 1
\]

\[
S_X(\omega) = \frac{1}{2T_b} + \frac{1}{4T_b} \sum_{n=\infty}^{\infty} e^{-j\pi T_b}
\]
\[
= \frac{1}{4T_b} + \frac{1}{4T_b} \sum_{n=\infty}^{\infty} e^{-j\omega n T_b}
\]
On-off Signaling

\[ S_X(\omega) = \frac{1}{4T_b} + \frac{2\pi}{4T_b^2} \sum_{n=-\infty}^{\infty} \delta \left( \omega - \frac{2\pi n}{T_b} \right) \]

\[ S_y(\omega) = \left| \frac{P(\omega)}{4T_b} \right|^2 \left[ 1 + \frac{2\pi}{T_b} \sum_{n=-\infty}^{\infty} \delta \left( \omega - \frac{2\pi n}{T_b} \right) \right] \]

For a half-width rectangular pulse,

\[ S_y(\omega) = \frac{T_b}{16} \sin^2 \left( \frac{\omega T_b}{4} \right) \left[ 1 + \frac{2\pi}{T_b} \sum_{n=-\infty}^{\infty} \delta \left( \omega - \frac{2\pi n}{T_b} \right) \right] \]

Note: 1. For some derivation, refer to the next three pages.
2. PSD is shown in Figure 7.7.
3. PSD consists of both a discrete part and a continuous part.

Some Derivation

(Lathi’s book, pp. 58-59, Example 2.12)

- An unit impulse train: \( \delta_{f_0}(t) = Comb_{f_0}(t) = g(t) \)

\[ g(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_b t} = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} e^{jn\omega_b t} \]

\[ = \frac{1}{T_b} + \frac{2}{T_b} \sum_{n=-\infty}^{\infty} \cos( n \omega_b t ) \]

\[ G(f) = FT\{g(t)\} = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta( f - nf_0 ) \]

\[ G(\omega) = FT\{g(t)\} = \frac{2\pi}{T_0} \sum_{n=-\infty}^{\infty} \delta( \omega - \frac{2\pi n}{T_0} ) \]

\[ \cong \omega_0 COMB_{\omega_0}(\omega) \]
Example 2.12

Find the exponential Fourier series and sketch the corresponding spectra for the impulse train $\delta_{T_0}(t)$ shown in Fig. 2.27.

Some Derivation…

- **Fact 1** (Lathi’s book, p.83, Eq. (3.20a, 3.20b):
  
  \[ 1 \leftrightarrow \delta(f) \]
  
  \[ e^{j2\pi f_0 t} \leftrightarrow \delta(f - f_b) \quad \left[ e^{j\omega_0 t} \leftrightarrow \delta(\omega - \omega_b) \right] \]

- **Fact 2:** Also, 
  
  \[ \delta(t) \leftrightarrow 1 \]
  
  \[ \delta(t - nT_b) \leftrightarrow e^{-jn\omega T_b} \]
Some Derivation …

- Fact 3:

\[
\sum_{n=-\infty}^{\infty} \delta(t-nT_b) = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} e^{jn\omega_b t} \quad \omega_b = \frac{2\pi}{T_b}
\]

\[
\uparrow \text{FT} \quad \uparrow \text{FT}
\]

\[
\sum_{n=-\infty}^{\infty} e^{-j\omega_b nT} = \omega_b \sum_{n=-\infty}^{\infty} \delta(\omega-n\omega_b)
\]

PSD of An On-off Signal

Figure 7.7 Power spectral density of an on-off signal
On-off Signaling

• This is reasonable since, as shown in Fig. 7.2, an on-off signal can be expressed as the sum of a polar and a periodic component

• Comment 1: For a given transmitted power, it is less immune to noise interference.
  – ::: Noise immunity \( \propto \) difference of amplitudes representing binary 0 and 1.
  – If a pulse of amplitude \( I \) or \( -I \) has energy \( E \), then a pulse of amplitude 2 has energy \( (2)^2E = 4E \).

\[
\begin{align*}
Q & \propto \frac{1}{T_b} \\
\therefore \quad \text{polar signal power} & = E \left( \frac{1}{T_b} \right) = \frac{E}{T_b}
\end{align*}
\]

• For on-off:

\[
\text{Power} = 4E \left( \frac{1}{2T_b} \right) = \frac{2E}{T_b}
\]

Power is now as large as twice of above.

• Comment 2: Not transparent

On-off Signaling …

• For polar: \( \frac{1}{T_b} \) digits are transmitted per second

\[
\therefore \quad \text{polar signal power} = E \left( \frac{1}{T_b} \right) = \frac{E}{T_b}
\]

• For on-off: \( \text{power} = 4E \left( \frac{1}{2T_b} \right) = \frac{2E}{T_b} \) Power is now as large as twice of above.

• Comment 2: Not transparent
Bipolar Signaling
(Pseudoternary or Alternate Mark Inverted (AMI))

A. $0 \leftrightarrow$ no pulse
   $1 \leftrightarrow p(t)$ or $-p(t)$ depending on whether the previous 1 was transmitted by $-p(t)$ or $p(t)$

B. $[p(t), 0, -p(t)]$: In reality, it is ternary signaling.

C. Merit: a dc null in PSD
demerit: not transparent

D. $R(0) = \lim_{N \to \infty} \frac{1}{N} \sum_{k} a_k^2 = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{2} (\pm 1)^2 + \frac{N}{2} (0) \right] = \frac{1}{2}$

$R(1) = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{4} (-1) + \frac{3N}{4} (0) \right] = -\frac{1}{4}$

$R(2) = \lim_{N \to \infty} \frac{1}{N} \left[ \frac{N}{8} (1) + \frac{N}{8} (-1) + \frac{5N}{8} (0) \right] = 0$

$R_n = \lim_{N \to \infty} \frac{1}{N} \sum_{k} a_k a_{k+n} = 0, n > 1$
Bipolar Signaling…

E. \[ S_y(\omega) = \left| P(\omega) \right|^2 S_x(\omega) \]

\[
= \frac{\left| P(\omega) \right|^2}{T_b} \left( \sum_{n=-\infty}^{\infty} R_n e^{-jn\omega T_b} \right) \quad (7.10b)
\]

\[
= \frac{\left| P(\omega) \right|^2}{T_b} \left( R_0 + 2 \sum_{n=1}^{\infty} R_n \cos n\omega T_b \right) \quad (7.10c)
\]

\[
= \frac{\left| P(\omega) \right|^2}{T_b} \left( \frac{1}{2} + 2 \cdot \left( -\frac{1}{4} \right) \cos \omega T_b \right)
\]

\[
= \frac{\left| P(\omega) \right|^2}{2 \cdot T_b} (1 - \cos \omega T_b) = \frac{\left| P(\omega) \right|^2}{T_b} \sin^2 \left( \frac{\omega T_b}{2} \right)
\]

Bipolar Sampling …

⇒ \[ S_y(\omega) = 0 \quad \text{As } \omega = 0 \text{ (dc) regardless of } P(\omega) \]

A dc null (desirable for ac coupling)

\[ \sin^2 \left( \frac{\omega T_b}{2} \right) = 0 \quad \text{at} \quad \omega = \frac{2\pi}{T_b}, \therefore f = \frac{1}{T_b} = R_b \]

⇒ bandwidth = \( R_b \text{Hz}, \) regardless of \( P(\omega) \)

For a half-width rectangular pulse, \[ \frac{T_b}{2}, P(t) = \text{rect} \left( \frac{t}{T_b/2} \right) \]

\[ P(\omega) = \frac{T_b}{2} \text{sinc} \left( \frac{\omega}{\frac{4}{T_b}} \right) \]
\[
S_y(\omega) = \frac{T_b}{4} \text{sinc}^2\left(\frac{\omega T_b}{4}\right) \sin^2\left(\frac{\omega T_b}{2}\right)
\]

Zero:
\[
\omega_1 = \frac{4n\pi}{T_b}, \quad \omega_2 = \frac{2m\pi}{T_b}
\]
\[
f_1 = 2nR_b, \quad f_2 = mR_b
\]

Essential bandwidth \(R_b\): Twice that of theoretical minimum bandwidth (channel b/w)

Half that of polar or on-off signaling

Table 3.1:
\[
\text{rect}\left[\frac{t}{\tau}\right] \leftrightarrow \tau \text{sinc}\left(\frac{\omega \tau}{2}\right) = \tau \text{sinc}\left(\frac{\omega}{2\tau}\right)
\]
Merits of Bipolar Signaling

1. Spectrum has a dc dull.
2. Bandwidth is not excessive.
3. It has single-error-detection capability.
   Since, a single detection error ⇒ a violation of the alternation pulse rule.

Demerits of Bipolar Signaling

1. Requires twice as much power as a polar signaling.
   Distinction between A, -A, 0
   vs.
   Distinction between A/2, -A/2
2. Not transparent
Pulse Shaping

\[ S_y(\omega) = \left| P(\omega) \right|^2 S_x(\omega) \]

\[ = \frac{|P(\omega)|^2}{T_b} (R_0 + 2 \sum_{n=1}^{\infty} R_n \cos n\omega T_b) \]

- Different signaling (line coding, \( S_x(\omega) \)) \( \Rightarrow \) diff. \( S_y(\omega) \)
- Different pulse shaping (\( P(\omega) \)) \( \Rightarrow \) diff. \( S_y(\omega) \)
- A more potent factor.

**Intersymbol Interference (ISI)**

- Time-limited pulses \( \Rightarrow \) not band-limited in frequency domain, truncation in frequency domain causes problem.
- Not time-limited, causes problem \( \Leftarrow \) band-limited signal

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Phase Shaping…

**Intersymbol Interference (ISI)**

- Whether we begin with time-limited pulses or band-limited pulses, it appears that ISI cannot be avoided.
  An inherent problem in the finite bandwidth transmission.
- Fortunately, there is a way to get away:
  be able to detect pulse amplitudes correctly.
Intersymbol Interference (ISI) in Detection

Nyquist Criterion for Zero ISI

- The first method proposed by Nyquist.
  \[ p(t) = \begin{cases} 
  1, & t = 0 \\
  0, & t = \pm n T_b, \quad T_b = \frac{1}{R_b} 
  \end{cases} \]  
  (7.22)

  \[ T_b : \text{separation between successive transmitted pulses} \]

- Example 6.1 (p.256)
  \[ p(t) = \sin c(2\pi Bt), \quad B = \frac{1}{2T_b} = \frac{1}{2} R_b \]

- Minimum bandwidth pulse that satisfies Nyquist Criterion Fig. 7.10 (b and c)
Nyquist Criterion for Zero ISI...

\[
p(t) = \sin c(\pi R_b t) = \begin{cases} 
1 & t = 0 \\
0 & t = \pm nT_b 
\end{cases} \quad (T_b = \frac{1}{R_b})
\]

\[
P(\omega) = \frac{1}{R_b} \text{rect} \left( \frac{\omega}{2\pi R_b} \right) \quad \text{bandwidth} = \frac{R_b}{2}
\]

Table 3.1:

\[
\frac{W}{\pi} \text{Sinc} (Wt) \leftrightarrow \text{rect} \left( \frac{\omega}{2W} \right)
\]
Problems of This Method

• Impractical
  1. \(-\infty < t < \infty\)
  2. It decays too slowly at a rate 1/t.
     \[\Rightarrow\text{Any small timing problem (Deviation) } \Rightarrow \text{ISI}\]
• Solution: Find a pulse satisfying Nyquist criterion (7.22), but decays faster than 1/t.
• Nyquist: Such a pulse requires a bandwidth
  \[k \cdot \frac{R_b}{2}, \quad 1 \leq k \leq 2\]

Let \(p(t) \leftrightarrow P(\omega)\)
  – The bandwidth of \(P(\cdot)\) is in \((R_b/2, R_b)\)
  – \(p(t)\) satisfies Nyquist criterion Equation (7.22)
• Sampling \(p(t)\) every \(T_b\) seconds by multiplying \(p(t)\) by an impulse train \(\delta_{T_b}(t)\).
  \[\Rightarrow \quad \overline{P}(t) = p(t) \delta_{T_b}(t) = \delta(t)\]
  \[\Downarrow \quad \text{FT}\]
  \[\Rightarrow \quad \frac{1}{T_b} \sum_{n = -\infty}^{\infty} P(\omega - n\omega_s) = 1\]
  Equation (6.4)
  \[G(\omega) = \frac{1}{T_s} \sum_{n = -\infty}^{\infty} P(\omega - n\omega_s)\]
Derivation

- $|P(\gamma)|$ is odd symmetric in $y_1-o-y_2$ system.
Derivation…

- The bandwidth of $P(\omega)$ is $\frac{\omega_p}{2} + \omega_x$
  - $\omega_x$: the excess bandwidth

- $r = \frac{\text{excess bandwidth}}{\text{theoretical minimum bandwidth}}$

  $\frac{\omega_x}{\omega_b} = \frac{2 \cdot \omega_x}{\omega_b} = \frac{1}{2} \quad 0 \leq r \leq 1$

- The bandwidth of $P(\omega)$ is $B_r = \frac{R_b}{2} + \frac{rR_b}{2} = \frac{(1 + r)R_b}{2}$

- $P(\omega)$, thus derived, is called a **Vestigial Spectrum**.

Realizability

- For a physically realizable system, $h(t)$, must be causal, i.e., $h(t) = 0$, for $t < 0$. (The n.c. & s.c.)

- In the frequency domain, the n.c. & s.c. is known as Paley-Wiener criterion

  $$\int_{-\infty}^{\infty} \ln |H(\omega)| \frac{1}{1 + \omega^2} d\omega < \infty$$

- Note that, for a physically realizable system, $H(\omega)$ may be zero at some discrete frequencies.
  - But, it cannot be zero over any finite band

- So, ideal filters are clearly unrealizable.
• From this point of view, the vestigial spectrum $P(\omega)$ is unrealizable.
• However, since the vestigial roll-off characteristic is gradual, it can be more closely approximated by a practical filter.
• One family of spectra that satisfies the Nyquist criterion is

$$
P(\omega) = \begin{cases} 
\frac{1}{2} \left(1 - \sin \left(\frac{\pi (\omega - \Omega_b)}{2\omega_x}\right)\right), & |\omega - \Omega_b| < \omega_x \\
0, & |\omega| > \frac{\Omega_b}{2} + \omega_x \\
1, & |\omega| < \frac{\Omega_b}{2} - \omega_x 
\end{cases}
$$

• Comment 1: Increasing $r$ (or $r$) improves $p(t)$. I.e., more gradual cutoff reduces the oscillatory nature of $p(t)$, $P(\omega)$ decays more rapidly
Realizability…

- Comment 2: As \( r = 1 \), i.e., \( \omega_s = \frac{\omega_b}{2} \)

\[
P(\omega) = \frac{1}{2} \left( 1 + \cos \frac{\omega}{2R_b} \right) \text{rect} \left( \frac{\omega}{4\pi R_b} \right)
\]

\[
= \cos^2 \left( \frac{\omega}{4R_b} \right) \text{rect} \left( \frac{\omega}{4\pi R_b} \right)
\]

- This characteristic is known as:
  
  The raised-cosine characteristic
  
  The full-cosine roll-off characteristic
Realizability…

\[ p(t) = R_b \frac{\cos(\pi R_b t)}{1 - 4 R^2_b t^2} \sin(\pi R_b t) \]

A. Bandwidth is \( R_b \) (r=1)
B. \( p(0) = R_b \)
C. \( p(t) = 0 \) at all the signaling instants & at points midway between all the signaling instants
D. \( p(t) \) decays rapidly as \( 1/t^3 \) relatively insensitive to derivations of \( R_b \), sampling rate, timing jitter and so on.

Realizability…

E. Closely realizable
F. Can also be used as a duobinary pulse.
G. \( H_c(\omega) \) : channel transfer factor
   \( P_t(k) \) : transmitted pulse
   \[ P_t(\omega) \cdot H_c(\omega) = P(\omega) \]
   Received pulse at the detector input should be \( P(\omega) \) [\( p(t) \)].
Signaling with Controlled ISI: Partial Response Signals

• The second method proposed by Nyquist to overcome ISI.

• Duobinary pulse: \( p(nT_b) = \begin{cases} 1 & n = 0, 1 \\ 0 & \text{for all other } n \end{cases} \)

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Figure 7.14 Communication using duobinary pulses.

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Signaling with Controlled ISI…

• Use polar signaling with this pulse
  
  \[ 1 \leftrightarrow p(t) \]
  
  \[ 0 \leftrightarrow -p(t) \]

• The received signal is sampled at \( t = nT_b \)
  
  \[ p(t) = 0 \quad \text{for all } n \quad \text{except } n = 0, 1 \]

• Clearly, such a pulse causes zero ISI with all the pulses except the succeeding pulses.
Signaling with Controlled ISI…

- Consider two such successive pulses located at 0 and $T_b$.
  - If both pulses are positive, the sample value at $t = T_b$ \( \Rightarrow 2 \).
  - If both pulses are negative, \( \Rightarrow -2 \)
  - If pulses are of opposite polarity, \( \Rightarrow 0 \)

- Decision Rule: If the sample value at $t = T_b$ is
  - Positive \( \Rightarrow 1, 1 \)
  - Negative \( \Rightarrow 0, 0 \)
  - Zero \( \Rightarrow 0, 1 \) or \( 1, 0 \)

---

Signaling with Controlled ISI…

- Figure 7.15

| Transmitted Seq. | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| Sample of $x(T_b)$ | 1 | 2 | 0 | 0 | 2 | 0 | -2 | -2 | 0 | 0 | 0 | 2 | 2 |
| Detected seq. | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
Signaling with Controlled ISI: Duobinary Pulses

- If the pulse bandwidth is restricted to be $\frac{R_b}{2}$, it can be shown that the $p(t)$ must be:

\[
p(t) = \frac{\sin(\pi R_b t)}{\pi R_b t(1 - R_b t)}
\]

\[
P(\omega) = \frac{2}{R_b} \cos \left( \frac{\omega}{2 R_b} \right) \text{rect} \left( \frac{\omega}{2 \pi R_b} \right) e^{-\frac{\omega}{2 R_b}}
\]

- Note: The raised-cosine pulse with $\omega_r = \frac{\omega}{2}, (r = 1)$, satisfies the condition (7.3b) to be signaling with controlled ISS – refer to Figure 7.13.
Use of Differential Coding

• For the controlled ISI method, 0-valued sample $\rightarrow$ 0 to 1 or 1 to 0 transition.
• An error may be propagated.
• Differential coding helps.
• In differential coding, “1” is transmitted by a pulse identical to that used for the previous bit. “0” is transmitted by a pulse negative to that used for the previous bit.
• Useful in systems that have no sense of absolute polarity. $\Rightarrow$ Fig 7.17

Differential Code

Figure 7.17 Differential code.
Scrambling

- **Purpose:** A scrambler tends to make the data more random by
  - Removing long strings of 1’s or 0’s
  - Removing periodic data strings.
  - Used for preventing unauthorized access to the data.

- **Example. (Structure)**

  Scrambler: \[ S \oplus D^3T \oplus D^5T = T \]
  
  \[ D^nT : \quad T \text{ delayed by } n \text{ units} \]
  
  \[ \oplus : \quad \text{Modulo 2 Sum} \]
  
  \[ 1 \oplus 1 = 0 \]
  
  \[ 0 \oplus 0 = 0 \]
Scrambling…

• Descrambler:

\[ R = T \oplus (D^3 \oplus D^5)T \]
\[ = [1 \oplus (D^3 \oplus D^5)]T \]
\[ = (1 \oplus F)T \]

\[ F = D^3 \oplus D^5 \]

Example 7.2

• The data stream 101010100000111 is fed to the scrambler in Fig. 7.19a. Find the scrambler output \( T \), assuming the initial content of the registers to be zero.

  – From Fig. 7.19a we observe that initially \( T = S \), and the sequence \( S \) enters the register and is returned as \( (D^3 \oplus D^5)S = FS \) through the feedback path. This new sequence \( FS \) again enters the register and is returned as \( F^2S \), and so on. Hence,

\[ T = S \oplus FS \oplus F^2S \oplus F^3S \oplus ... \]  \hspace{1cm} (7.41)
\[ = (1 \oplus F \oplus F^2 \oplus F^3 \oplus ...)S \]
Example 7.2 …

- Recognizing that \( F = D^3 \oplus D^5 \)
  We have
  \[
  F^2 = (D^3 \oplus D^5)(D^3 \oplus D^5) = D^6 \oplus D^{10} \oplus D^8 \oplus D^8
  \]

- Because modulo-2 addition of any sequence with itself is zero, \( D^8 \oplus D^8 = 0 \) and
  \( F^2 = D^6 \oplus D^{10} \)

- Similarly,
  \[
  F^3 = (D^6 \oplus D^{10})(D^3 \oplus D^5) = D^9 \oplus D^{11} \oplus D^{11} \oplus D^{15}
  \]

…… and so on ……

Hence,
\[
T = (1 \oplus D^3 \oplus D^5 \oplus D^6 \oplus D^8 \oplus D^{10} \oplus D^{11} \oplus D^{12} \oplus D^{13} \oplus D^{15} \oplus \ldots)S
\]

Because \( D^nS \) is simply the sequence \( S \) delayed by \( n \) bits, various terms in the preceding equation correspond to the following sequences:
Example 7.2 …

\[ S = 10101010000111 \]
\[ D^3S = 00010101010000111 \]
\[ D^5S = 0000010101010000111 \]
\[ D^6S = 00000010101010000111 \]
\[ D^9S = 000000000101010000111 \]
\[ D^{10}S = 0000000000101010000111 \]
\[ D^{11}S = 0000000000010101010000111 \]
\[ D^{12}S = 00000000000010101010000111 \]
\[ D^{13}S = 000000000000010101010000111 \]
\[ D^{15}S = 0000000000000001010101010000111 \]
\[ T = 1011100011101001 \]

Note that the input sequence contains the periodic sequence 10101010…, as well as a long string of 0’s.

The scrambler output effectively removes the periodic component as well as the long strings of 0’s.

The input sequence has 15 digits. The scrambler output up to the 15th digit only is shown, because all the output digits beyond 15 depend on the input digits beyond 15, which are not given.

We can verify that the descrambler output is indeed S when this sequence T is applied at its input.
Regenerative Repeater

• Three functions:

1. Reshaping incoming pulses using an equalizer.
2. Extracting timing information
   Required to sample incoming pulses at optimum instants.
3. Making decision based on the pulse samples
Preamplifier and Equalizer

- A pulse train is attenuated and distorted by transmission medium, say, dispersion caused by an attenuation of high-frequency components.
- Restoration of high frequency components \( \Rightarrow \) increase of channel noise
- Fortunately, digital signals are more robust.
  - Considerable pulse dispersion can be tolerated
- Main concern:
  Pulse dispersion \( \Rightarrow \) ISI
  \( \Rightarrow \) increase error probability in detection

Zero-Forcing Equalizer

- Detection decision is based solely on sample values.
  \( \Rightarrow \) No need to eliminate ISI for all \( t \).
  All that is needed is to eliminate or minimize ISI at their respective sampling instants only.
- This could be done by using the transversal-filter equalizer, which forces the equalizer output pulse to have zero values at the sampling (decision-making) instant.
Zero-Forcing Equalizer …

• Let \( c_0 = 1 \)
  \[ c_k = 0 \quad \forall k \neq 0 \]  
  tap setting

\[ \Rightarrow P_0(t) = P_r(t - NT_b) \]
\[ P_0(t) = P_r(t) \quad \text{If ignore delay.} \]

• Fig 7.21 (b) indicates a problem:
  \( a_1, a_{-1}, a_2, a_{-2}, \ldots \) are not negligible due to dispersion.

• Now, want to force \( a_1 = a_{-1} = a_2 = a_{-2} = \ldots = 0 \)

• Consider \( c_k \)'s assume other values (other tap setting).
Zero-Forcing Equalizer …

• Consider \( c_k \)'s assume other values (other tap setting).
  \[
p_0(t) = \sum_{n=-N}^{N} c_n p_r(t-nT_b)
\]
  \[
p_0(kT_b) = \sum_{n=-N}^{N} c_n p_r[(k-n)T_b], \quad k = 0, \pm 1, \pm 2, \ldots
\]

• For simplicity of notation:
  \[
p_0(k) = \sum_{n=-\infty}^{\infty} C_n p_r(k-n)
\]

• Nyquist Criterion:
  \[
p_0(k) = 0 \quad \forall k \neq 0
\]
  \[
p_0(k) = 1 \quad \text{for} \quad k = 0
\]

Zero-Forcing Equalizer …

• \( \Rightarrow \) A set of infinitely many simultaneous equations:
  \[
  2N+1: c_n \text{’s}
  \]
  Impossible to solve this set of equations.

• If, however, we specify the values of \( p_0(k) \) only at \( 2N+1 \) points:
  \[
  p_0(k) = \begin{cases} 
  1 & k = 0 \\
  0 & k = \pm 1, \pm 2, \ldots, \pm N 
  \end{cases}
  \]
  then a unique solution exists.
Zero-Forcing Equalizer …

- Meaning: Zero ISI at the sampling instants of N preceding and N succeeding pulses.

- Since pulse amplitude decays rapidly, ISI beyond the $N$th pulse is not significant for $N>2$ in general.

\[ \begin{bmatrix} p_0(-N) \\ \vdots \\ p_0(1) \\ p_0(0) \\ \vdots \\ p_0(N) \end{bmatrix} = \begin{bmatrix} p_1[0] & p_1[-1] & \cdots & p_1[-2N] \\ p_1[1] & p_1[0] & \cdots & p_1[-2N-1] \\ \vdots & \vdots & \ddots & \vdots \\ p_1[N-1] & p_1[N-2] & \cdots & p_1[-N-1] \\ p_1[N] & p_1[N-1] & \cdots & p_1[-N] \\ \vdots & \vdots & \ddots & \vdots \\ p_1[N+1] & p_1[N] & \cdots & p_1[-N+1] \end{bmatrix} \begin{bmatrix} c_{-N} \\ c_{-N+1} \\ \vdots \\ c_{-1} \\ c_0 \\ c_1 \\ \vdots \\ c_{N-1} \\ c_N \end{bmatrix} \]

(7.45)

The tap-gain $c_i$'s can be obtained by solving this set of equations.
**Zero-Forcing Equalizer**

**Example 7.3** For the received pulse $p_i(t)$ in Fig. 7.21b, let

$$a_0 = p_i[0] = 1$$
$$a_1 = p_i[1] = -0.3, \quad a_2 = p_i[2] = 0.1$$
$$a_{-1} = p_i[-1] = -0.2, \quad a_{-2} = p_i[-2] = 0.05$$

Design a three-tap ($N = 1$) equalizer.

Substituting the preceding values into Eq. (7.45), we obtain

$$
\begin{bmatrix}
0 \\
1 \\
0 \\
\end{bmatrix} = \begin{bmatrix}
1 & -0.2 & 0.05 \\
-0.3 & 1 & -0.2 \\
0.1 & -0.3 & 1 \\
\end{bmatrix} \begin{bmatrix}
c_{-1} \\
c_0 \\
c_1 \\
\end{bmatrix}
$$

Solution of this set yields $c_{-1} = 0.210, \quad c_0 = 1.13,$ and $c_1 = 0.318.$ This tap setting assures us that $p_i[0] = 1$ and $p_i[-1] = p_i[1] = 0.$ The output $p_o(t)$ is sketched in Fig. 7.21c.

---

**Eye Diagram**

- A convenient way to study ISI on an oscilloscope
- Figure

---

Dr. Shi  
Digital Communications
Timing Extraction

• The received signal needs to be sampled at precise instants.
• Timing is necessary.
• Three ways for synchronization:
  1. Derivation from a primary or a secondary standard (a master timing source exists, both transmitter and receiver follow the master).
  2. Transmitting a separate synchronizing signal (pilot clock)
  3. Self synchronization (timing information is extracted from the received signal itself).

Timing Extraction …

• Way 1: Suitable for large volumes of data high speed comm. Systems.
  → High cost.
• Way 2: Part of channel capacity is used to transmit timing information.
  → Suitable for a large available capacity.
• Way 3: Very efficient.
• Examples:
  – On-off signaling (decomposition: Fig. 7.2)
  – Bipolar signaling → rectification on-off signaling
Timing Jitter

- Small random deviation of the incoming pulses from their ideal locations.
- Always present, even in the most sophisticated communication systems.
- Jitter reduction is necessary about every 200 miles in a long digital link to keep the maximum jitter within reasonable limits.
- Figure 7.23
Detection – Error Probability

- Received signal = desired + AWGN
  Example: Polar signaling and transmission (Fig. 7.24, the next slide)
  \( A_p \): Peak signal value
- Polar: Instead of \( \pm A_p \), received signal = \( \pm A_p + n \)
- Since symmetry, the detection threshold = 0.
  - If sample value > 0, \( \Rightarrow \) “1”
  - If sample value < 0, \( \Rightarrow \) “0”
  - Error in detection may take place

**Figure 7.24** Error probability in threshold detection.
Error Probability for Polar Signal

- With respect to Figure 7.24
  \[ P(\epsilon / 0) = \text{probability that } n > A_p \]
  \[ P(\epsilon / 1) = \text{probability that } n < -A_p \]

\[ n: \text{AWGN} \]
\[ p(n) = \frac{1}{\sqrt{2\pi} \sigma_p} e^{-\frac{1}{2} \left( \frac{n}{\sigma_p} \right)^2} \]

\[ \Rightarrow P(\epsilon / 0) = \frac{1}{\sigma_p \sqrt{2\pi}} \int_{A_p}^{\infty} e^{-\frac{1}{2} \left( \frac{n}{\sigma_p} \right)^2} dn \]
\[ = \frac{1}{\sqrt{2\pi}} \int_{\frac{A_p}{\sigma_p}}^{\infty} e^{-x^2} dx = Q \left( \frac{A_p}{\sigma_p} \right) \]

Error Probability for Polar Signal...

where \[ Q(y) = \frac{1}{\sqrt{2\pi}} \int_{y}^{\infty} e^{-\frac{1}{2} x^2} dx \]

- Similarly,
  \[ P(\epsilon/1) = Q \left( \frac{A_p}{\sigma_p} \right) \quad \text{Symmetric} \]

\[ P(\epsilon) = P(\epsilon/0) + P(\epsilon/1) \]
\[ = P(\epsilon/0)P(0) + P(\epsilon/1)P(1) \]
\[ = 0.5 \left[ P(\epsilon/0) + P(\epsilon/1) \right] = Q \left( \frac{A_p}{\sigma_p} \right) \quad \text{Here, assume } P(0) = P(1) = 0.5 \]

- \( Q(x) \): Complementary error function: \( \text{erfc} (x) \)
  \[ Q(x) \equiv \frac{1}{x \sqrt{2\pi}} \left( 1 - 0.7 \right) e^{-\frac{x^2}{2}}, \quad x > 2 \]

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Error Probability for Polar Signal…

- If \( \frac{A_p}{\sigma_n} = k \), \( P(\varepsilon/0) = P(\varepsilon/1) = P(\varepsilon) = Q(k) \)

<table>
<thead>
<tr>
<th>( k )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q(k) = P(\varepsilon) )</td>
<td>0.1587</td>
<td>0.0227</td>
<td>0.00135</td>
<td>3.16×10^-4</td>
<td>2.87×10^-5</td>
<td>9.9×10^-9</td>
</tr>
</tbody>
</table>

e.g. \( 3.16\times10^{-3} \) \( \Rightarrow \) one of \( 3.16\times10^{-3} \) pulses will be possibly detected wrongly

Error Probability for On-off Signals

- In this case, we have to distinguish between \( A_p \) and 0.
- Threshold: \( \frac{1}{2} A_p \)
- Error Probabilities:

\[
P(\varepsilon/0) = \text{prob. of } n > \frac{1}{2} A_p = Q \left( \frac{A_p}{2\sigma_n} \right)
\]

\[
P(\varepsilon/1) = \text{prob. of } n < -\frac{1}{2} A_p = Q \left( \frac{A_p}{2\sigma_n} \right)
\]

\[
P(\varepsilon) = \frac{1}{2} \left[ P(\varepsilon/0) + P(\varepsilon/1) \right] = Q \left( \frac{A_p}{2\sigma_n} \right)
\]

\( P(\varepsilon) > P(\varepsilon) \) in the case of polar signaling.
Error Probability for Bipolar Signals

1 \leftrightarrow A_p or -A_p

0 \leftrightarrow 0v

\Rightarrow \quad \text{If the detected sample value: } 
\quad \in \left( -\frac{A_p}{2}, \frac{A_p}{2} \right) \leftrightarrow 0

\text{otherwise } \leftrightarrow 1

\begin{align*}
P(\varepsilon/0) &= \text{prob} \left( n > \frac{A_p}{2} \right) \\
&= \text{prob} \left( n > \frac{A_p}{2} \right) + \text{prob} \left( n < -\frac{A_p}{2} \right) \\
&= 2 \text{prob} \left( n > \frac{A_p}{2} \right) = 2Q \left( \frac{A_p}{2\sigma_n} \right)
\end{align*}
Error Prob. for Bipolar Signals…

\[ P(\varepsilon/1) = \text{prob}\left( n < -\frac{A_p}{2}\right) \text{ when positive pulse used} \]

or \( \text{prob}\left( n > \frac{A_p}{2}\right) \text{ when negative pulse used} \)

\[ = Q\left( \frac{A_p}{2\sigma_n} \right) \]

\[ P(\varepsilon) = P(\varepsilon/0) P(0) + P(\varepsilon/1) P(1) = \frac{1}{2} [P(\varepsilon/0) + P(\varepsilon/1)] \]

\[ = 1.5Q\left( \frac{A_p}{2\sigma_n} \right) > P(\varepsilon) \text{ for on-off signaling} \]

Detection Error Probability…

- Summary, polar is the best, \( Q\left( \frac{A_p}{\sigma_n} \right) \)  "0" and "1" Equally Likely Assumption

  on-off is in middle, \( Q\left( \frac{A_p}{2\sigma_n} \right) \)

  bipolar is the worst, \( 1.5Q\left( \frac{A_p}{2\sigma_n} \right) \)

- Another factor: \( P(\varepsilon) \) decreases exponentially with the signal power.
Comparison between Three Line Codes

• To obtain: \( P(\varepsilon) = 0.286 \times 10^{-6} \)

• We need:
  \[
  \frac{A_p}{\sigma_n} = 5 \quad \text{for polar case} \quad \therefore Q(5) = 0.286 \times 10^{-6}
  \]
  \[
  \frac{A_p}{\sigma_n} = 10 \quad \text{for on-off case} \quad \therefore P(\varepsilon) = Q\left( \frac{A_p}{2\sigma_n} \right)
  \]
  \[
  \frac{A_p}{\sigma_n} = 10.16 \quad \text{for bipolar case} \quad \therefore P(\varepsilon) = 1.5Q\left( \frac{A_p}{2\sigma_n} \right) = 0.286 \times 10^{-6}
  \]
  \[
  \therefore \frac{A_p}{2\sigma_n} = 5.08
  \]

Comparison between Three Line Codes...

• For the same error rate, required SNR \( \left\{ \frac{A_p}{\sigma_n} \right\} \), Polar < On-off < Bipolar \quad \text{(from previous example)}

• For the same SNR \( \left\{ \frac{A_p}{\sigma_n} \right\} \), the caused error rate
  \[
  \text{Polar < On-off < Bipolar} \quad \text{(from next example)}
  \]

• Polar is most efficient in terms of SNR vs. error rate

• Between, on-off and bipolar:
  Only 16% improvement of on-off over bipolar

• Performance of bipolar case \( \approx \) performance of on=off case in terms of error rate.
Example 7.4

a) Polar binary pulses are received with peak amplitude $A_p = 1\,\text{mV}$. The channel noise rms amplitude is 192.3 $\mu\text{V}$. Threshold detection is used, and 1 and 0 are equally likely.

b) Find the error probability for (i) the on-off case (ii) the bipolar case if pulses of the same shape as in part (a) are used, but their amplitudes are adjusted so that the transmitted power is the same as in part (a).

Example 7.4 …

a) For the polar case

$$\frac{A_p}{\sigma_n} = \frac{10^{-3}}{192.3(10^{-6})} = 5.2$$

From Table 10.2, we find

$$P(\epsilon) = Q(5.2) = 0.9964 \times 10^{-7}$$

b) Because half the bits are transmitted by no-pulse, there are, on the average, only half as many pulses in the on-off case (compared to the polar). Now, doubling the pulse energy is accomplished by multiplying the pulse by $\sqrt{2}$.
Example 7.4 …

Thus, for on-off $A_p$ is $\sqrt{2}$ times the $A_p$ in the polar case.

• Therefore, from Equation (7.53)

$$P(\varepsilon) = Q\left(\frac{A_p}{2\sigma_n}\right) = Q(3.68) = 1.66 \times 10^{-4}$$

• As seen earlier, for a given power, the $A_p$ for both the on-off and the bipolar cases are identical. Hence, from equation (7.54)

$$P(\varepsilon) = 1.5Q\left(\frac{A_p}{2\sigma_n}\right) = 1.749 \times 10^{-4}$$

M-ary Communication

• Digital communications uses only a finite number of symbols

• Information transmitted by each symbol increases with $M$.

$$I_M = \log_2 M \quad \text{bits} \quad I_M : \text{information transmitted by an M-ary symbol}$$

• Transmitted power increases as $M^2$.

i.e., to increase the rate of communication by a factor of $\log_2 M$, the power required increases as $M^2$. 

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M-ary Communication…

• Most of the terrestrial digital telephone network: Binary
• The subscriber loop portion of the integrated services digital network (ISDN) uses the quarternary code 2BIQ shown in Figure 7.25

Figure 7.25 4-ary multi-amplitude signal, code x 8 1 (a)
Pulse Shaping in Multi-amplitude Case

- Nyquist Criterion can be used for M-ary case
- Controlled ISI
- Figure 7.25: One possible M-ary Scheme
- Another Scheme: Use M orthogonal pulses:
  \[ \phi_1(t), \phi_2(t), \ldots, \phi_M(t) \]
- Definition:
  \[
  \int_0^{T_b} \phi_i(t) \phi_j(t) = \begin{cases} 
  c & i = j \\
  0 & i \neq j 
  \end{cases}
  \]

Pulse Shaping in Multi-amplitude Case

- Figure 7.26: One example in M orthogonal pulses:
  \[ \phi_k(t) = \begin{cases} 
  \sin \left( \frac{2\pi kt}{T_b} \right) & 0 < t < T_b \quad k = 1, 2, \ldots, M \\
  0 & \text{otherwise} 
  \end{cases} \]
- In the set, pulse frequency:
  \[
  k \frac{1}{T_b} : \frac{1}{T_b}, \frac{2}{T_b}, \ldots, \frac{M^*}{T_b} \quad \text{M times that of the binary scheme}
  \]
Pulse Shaping in Multi-amplitude Case

- In general, it can be shown that the bandwidth of an orthogonal M-ary scheme is M times that of the binary scheme.
  - In an M-ary orthogonal scheme, the rate of communication is increased by a factor of $\log_2 M$ at the cost of an increase in transmission bandwidth by a factor of M.
Digital Carrier Systems

• So far: Baseband digital systems
  – Signals are transmitted directly without any shift in frequency.
  – Suitable for transmission over wires, cables, optical fibers.
• Baseband signals cannot be transmitted over a radio link or satellites.
  – Since, needs impractically large antennas
  – Modulation: Shift of signal spectrum to higher frequencies

Digital Carrier Systems…

• A spectrum shift to higher frequencies is also required when transmitting several messages simultaneously by sharing the large bandwidth of the transmission medium,
• FDM: Frequency-division Multiplexing.
  (FDMA: Frequency-division Multiplexing Access)
Several Types of Modulation

- **Amplitude-Shift Keying (ASK)**, also known as on-off keying (OOK)

\[ m(t) \cos(\omega_c t) : \quad m(t) : \text{on-off base band signal (modulating signal)} \]
\[ \cos(\omega_c t) : \text{carrier} \]

- **Phase-Shift Keying (PSK)**

\[ 1 \leftrightarrow p(t) \cos(\omega_c t), 0 \leftrightarrow -p(t) \cos(\omega_c t) = p(t) \cos(\omega_c t - \pi) \]

**Modulation Types…**

- **Phase-Shift Keying (PSK)**

\[ m(t) : \text{polar Signal} \]

\[ 1 \leftrightarrow p(t) \cos(\omega_c t), 0 \leftrightarrow -p(t) \cos(\omega_c t) = p(t) \cos(\omega_c t - \pi) \]
Modulation Types…

- **Frequency-Shift Keying (FSK)**

  - Frequency is modulation by the base band signal.

  \[ 1 \leftrightarrow \omega_c, 0 \leftrightarrow \omega_c \]

  - Information bit resides in the carrier frequency.

PSD of ASK, PSK & FSK
FSK

• FSK signal may be viewed as a sum of two interleaved ASK signals, one with a modulating frequency $\omega_{c_0}$, the other, $\omega_{c_1}$
  – Spectrum of FSK = Sum of the two ASK
  – Bandwidth of FSK is higher than that of ASK or PSK

Demodulation

A. Review of Analog Demodulation

• Double-sideband Suppressed Carrier (DSB-SC) Modulation

\[ m(t) \leftrightarrow M(\omega) \quad \text{message signal} \]
\[ \cos(\omega_c t) \leftrightarrow \frac{1}{2} \left[ \delta(\omega + \omega_c) + \delta(\omega - \omega_c) \right] \quad \text{carrier} \]
\[ m(t)\cos(\omega_c t) \leftrightarrow \frac{1}{2} \left[ M(\omega + \omega_c) + M(\omega - \omega_c) \right] \quad \text{modulated signal.} \]
Figure 4.1 (a), (b), (c) DSB-SC Modulation

Figure 4.1 (e) and (d) Using a carrier for exactly the same frequency & pulse

DSB-SC Demodulation

- Demodulation:
  - Synchronization Detection
  - Coherent Detection
- Figure 4.1 (e) and (d)
DSB-SC Demodulation …

\[
\left[ m(t) \cos(\omega_c t) \right] \cos(\omega_c t) = e(t) = \frac{1}{2} \left[ m(t) + m(t) \cos(2\omega_c t) \right]
\]

\[
E(\omega) = \frac{1}{2} M(\omega) + \frac{1}{4} \left[ M(\omega + 2\omega_c) + M(\omega - 2\omega_c) \right]
\]

After LPF \( \Rightarrow \frac{1}{2} M(\omega) \)

\[
\frac{1}{2} m(t)
\]

Figure 4.1 (d), (e) **DSB-SC demodulation**

![DSB-SC Demodulation Diagram](image)

**Figure 4.1** DSB-SC modulation and demodulation.
Amplitude Modulation (AM)

\[ \varphi_{AM}(t) = A \cos(\omega_c t) + m(t) \cos(\omega_c t) \]
\[ = [A + m(t)] \cos(\omega_c t) \]
\[ \varphi_{AM}(t) \leftrightarrow \frac{1}{2} \left[ M(\omega + \omega_c) + M(\omega - \omega_c) \right] \]
\[ + \pi \cdot A \left[ \delta(\omega + \omega_c) + \delta(\omega - \omega_c) \right] \]

Demodulation of AM Signal

• Motivation:
  – Requirements of a carrier in frequency and phase synchronism with carrier at the transmitter is too sophisticated and quite costly
  – Sending the carrier \( \Rightarrow \) Eases the situation
    requires more power in transmission suitable for broadcasting
• AM Signal and its Envelope

![AM Signal and its Envelope Diagram](image)

• Demodulation of AM signals
  
  – Rectifier Detection: Figure 4.11  
  Output: \( \frac{1}{\pi} m(t) \)

  – Envelope Detector: Figure 4.12  
  Output: \( A + m(t) \)
Figure 4.11 Rectifier detector for AM.

Figure 4.12 Envelope detector for AM.
B. Back to Digital Communication System: Demodulation

- Demodulation of Digital-Modulated Signals is similar to Demodulation of Analog-Modulated Signals

- Demodulation of ASK
  - Synchronous (Coherent) Detection
  - Non-coherent Detection: Say, envelope detection

Demodulation of PSK

- Cannot be demodulated non-coherently (envelope detection).
  - Since, envelope is the same for 0 and 1
- Can be demodulated coherently.
- PSK may be demodulated non-coherently: Differential coherent PSK (DPSK)
Differential Encoding

• Differential coding:
  – 1 $\leftrightarrow$ encoded by the same pulse used to encode the previous data bit $\rightarrow$ no transition
  – 0 $\leftrightarrow$ encoded by the negative of pulse used to encode the previous data bit $\rightarrow$ transition

• Figure 7.30 (a)

• Modulated signal consists of pulses $\pm A\cos(\omega_c t)$ with a possible sign ambiguity

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Differential Encoding…

- $T_b$: one-bit interval
- If the received pulse is identical to the previous pulse $\leftrightarrow 1$,  
  $$Y(t) = A^2 \cos^2(\omega_c t) = \frac{A^2}{2} [1 + \cos(2\omega_c t)]$$  
  $$z(t) = \frac{A^2}{2}$$
- If 0 is transmitted  
  $$Y(t) = -A^2 \cos^2(\omega_c t) = -\frac{A^2}{2} [1 + \cos(2\omega_c t)]$$  
  $$z(t) = -\frac{A^2}{2}$$

Demodulation of FSK

- FSK can be viewed as two interleaved ASK signals with carrier frequency $\omega_c$ and $\omega_c'$, respectively.
- Hence, FSK can be detected by non-coherent detection (envelope) or coherent detection technique
Demodulation of FSK

• In (a), Non-coherent detection (envelope),
  – $H_0(\omega)$
  $\omega_{c_0}$
  Turned to respectively
  – $H_1(\omega)$
  $\omega_{c_1}$

• Comparison: above or bottom
  Whose output is large $\Rightarrow$ 0 or 1
Digital Multiplexing

- TDM: Time-division Multiplexing
  - Several low-bit-rate signal can be multiplexed to form one high-bit-rate signal to be transmitted over a high-frequency medium.
- If all tributaries have the same bit rate, multiplexing can be done on
  1. A bit-by-bit basis (bit interleaving)
  2. A word-by-word basis (word interleaving)
Digital Multiplexing

- North American Digital Hierarchy: bit interleaving
- DSI Signal, SONET-formatted signal: word interleaving
- If the bit rates of incoming channels are not identical, the high-bit-rate channels are allocated proportionally more slots.
- At the receiving end, incoming bit stream must be divided and distributed to appropriate output channel.
  - The receiving terminal needs to identify each bit correctly
  - Framing and synchronization bits added to bit stream
  - Overhead bits

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Digital Multiplexing

- Interleaving channel having different bit rate
- Alternate scheme for (c)

Figure 7.33 Time-division multiplexing of digital signals.

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Digital Multiplexing

• Figure 7.33 (C):
  – Channel B, C, D having the same bit rate: R
  – Channel A having the bit rate: 3R
  – In 6-bit-group, 3 bits in A
    1 bit in B, C, D
• Figure 7.33 (D): An alternate scheme

Digital Hierarchy

• North American Digital Hierarchy (AT & T System): North America & Japan
• Figure 7.36 (next slide)

64 kbps x 24 = 1.536 Mbps
1.544 mbps x 4 = 6.176 Mbps
6.312 mbps x 7 = 44.184 Mbps
44.736 mbps x 3 = 134.208 Mbps
CCITT

- Consultative Committee on International Telephony and Telegraphy:
  - Europe and the rest of the world.
CCITT

Figure 7.37 Digital hierarchy, CCITT recommendation.