1. (2 points) Consider a binary random process that, with probability 1/3, is a stationary Markov chain with transition probabilities (from 0 to 1 and from 1 to 0) both equal to 1/3, and with probability 2/3 is a memoryless Ber(1/2) process.
   a. Calculate the entropy rate.
   b. Suggest a compression scheme that achieves the entropy rate.

2. (1 point) The transmitter chooses an hour of the day and the recipient receives information about the correct hour with probability 1/2 and about the previous or next hour with equal probability 1/4. No information about the day information is sent and hence the hour alphabet is \{0,...,23\} with 23+1=0. What is the capacity of this channel?

3. (1 point) Are the sequences \( x^4 = (0,0,0,1) \) and \( y^4 = (0,e,1,1) \) jointly typical for \( \epsilon = 0.1 \) if the input distribution is Ber(0.5) and the channel is an erasure channel with erasure probability 0.3? Explain.

4. (2 points) Consider a memoryless source \( V^m \) and a memoryless channel \( p(y|x) \), used \( n \) times, which has capacity \( C \). Prove that, even with feedback, the condition \( n/m > H(V)/C \) is necessary to ensure vanishing probability of error as \( m \) and \( n \) go to infinity. For this purpose, use the data processing inequality, the Fano inequality and steps similar to the converse on the channel capacity.

5. (1 point) Given an i.i.d. sequence \( X^n \) of variables with zero mean and power 1, what are the probabilities of the events \( \frac{1}{n} \sum_{i=1}^{n} X_i^2 < 1.1 \) and \( \frac{1}{n} \sum_{i=1}^{n} X_i^2 < 0.9 \) when \( n \) is large?

6. (1 point) Find the maximum entropy distribution \( p(x) \) with \( \mathcal{X} = \{0,1,2\} \) and \( E[X^2] = 1 \).

7. (2 points) Prove that for any analog source with differential entropy \( h \), the mean squared distortion must satisfy the inequality \( D \geq \frac{2^{2h}}{2\pi e} 2^{-2R} \) if \( R \) is the number of available bits per source sample.

8. (3 points) Consider a ternary source with alphabets \( \mathcal{X} = \{0,?,1\} \) and pmf \( p(?) = q \) and \( p(0) = p(1) = (1 - q)/2 \)
   a. Calculate the entropy of this source.
   b. Calculate the rate-distortion function for this source with reproduction alphabet \( \hat{\mathcal{X}} = \{0,1\} \), under distortion metric \( d(x, \hat{x}) \) defined as \( d(i, i) = 0 \) for \( i \in \{0,?,1\} \); \( d(0,1) = d(1,0) = \infty \); and \( d(?, i) = 0 \) for \( i \in \{0,1\} \).